

Extended Parameterisations for MSTW PDFs and their effect on Lepton Charge Asymmetry from W Decays

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Abstract

We investigate the effect of extending the standard MSTW parameterisation of input parton distribution functions (PDFs) using Chebyshev polynomials, rather than the usual expressions which involve a factor of the form $(1 + \epsilon x^{0.5} + \gamma x)$. We find evidence that, at most, four powers in the polynomial are sufficient for extremely high precision. Applying this to valence and sea quarks, the gluon already being sufficiently flexible and needing only two powers, we find an improvement in the global fit, but a significant change only in the small- x valence up-quark PDF, u_V . We investigate the effect of also extending, and making more flexible, the ‘nuclear’ correction to deuteron structure functions. We show that the extended ‘Chebyshev’ parameterisation results in an improved stability in the deuteron corrections that are required for the best fit to the ‘global’ data. The resulting PDFs have a significantly, but not dramatically, altered valence down-quark distribution, d_V . It is shown that, for the extended set of MSTW PDFs, their uncertainties can be obtained using 23, rather than the usual 20, orthogonal ‘uncertainty’ eigenvectors. This is true both without and with extended deuteron corrections. Since the dominant effect is on the valence quarks, we present a detailed study of the dependence of the valence–sea separation on the predictions for the decay lepton charge asymmetry which results from W^\pm production at the LHC, illustrating the PDFs and the x range probed for different experimental scenarios. We show that the modified MSTW PDFs make significantly improved predictions for these data at the LHC, particularly for high values of the p_T cut of the decay lepton. However, this is a special case, since the asymmetry is extremely sensitive to valence–sea details, and in particular to the combination $u_V - d_V$ of valence PDFs for $x \sim M_W/\sqrt{s}$ at low lepton rapidities. We show that the predictions for a wide variety of total cross sections are very similar to those obtained using the MSTW2008 PDFs, with changes being much smaller than the PDF uncertainties.

¹Work mainly done while at University College London.

1 Introduction

In the determination of the parton distribution functions (PDFs) of the proton from fitting to the available deep-inelastic and related hard-scattering data, a long-standing question is the extent to which the limitations of a fixed form of the input parameterisations affect the best fit and the uncertainty of the resulting PDFs. It is certainly the case that various groups performing ‘global’ PDF analyses have had to introduce new parameters to facilitate a good-quality fit to some new data, either because they probe a new PDF combination or a new kinematic range, or simply because new data are much more precise than previous measurements. This has resulted in most groups performing fits to a wide variety of data sets [1–4] using about 4-6 free parameters for each type of parton².

The NNPDF group [6] circumvents this issue by using effectively an extremely large and flexible parameterisation, but in order to avoid fitting all the fluctuations in data, they must split data into training and validation sets and have algorithms which determine the methods of both convergence and ‘stopping’. This means they do not have an easily identifiable ‘best fit’ and it is very difficult to compare the sources of their PDF uncertainty to those for other groups. Indeed, there is clear sensitivity to their convergence and ‘stopping’ algorithms. It is hypothesised that the lack of parameter flexibility is part of the reason for the need for a ‘tolerance’, or the use of ‘ $\Delta\chi^2 > 1$ ’, to obtain uncertainties in the MSTW fits. But studies so far suggest that while this is probably a component, it is not all, or even the dominant reason for this inflation of $\Delta\chi^2$ [7]. It is possible that the reason why (for well-constrained PDFs) the NNPDF uncertainty is very similar to that of MSTW and CTEQ, is partially due to the algorithms for fitting and ‘stopping’ inherently introducing something similar to the $\Delta\chi^2$, on account of some level of incompatibility between data sets (see e.g. [8]).

So far, there has been surprisingly little investigation of the change of the form of input parameterisations on the uncertainties of PDF sets based on a best fit and expansion about this central PDF set. There are studies by Pumplin [7], by Glazov, Moch and Radescu [9], and one sentence in the MSTW conference proceedings [10].

In this article we investigate the effect of extending the MSTW parameterisation of the input PDFs by changing the interpolating polynomial ($1 + \epsilon x^{0.5} + \gamma x$), which was introduced for separate up and down valence and for sea quarks in [11], to an term including up to an n^{th} -order Chebyshev polynomial. First, we investigate the most appropriate order of polynomial to use such that sufficient flexibility is achieved, but not so much that one is in danger of fitting fluctuations in the data. To study this, general functions of a suitable shape are generated, and pseudo-data are fitted. We conclude that up to a 4th-order polynomial should generally be adequate. We then try using this type of polynomial in fits to real data, first for the two valence distributions, but also additionally for the sea distribution. We see a significant improvement in the χ^2 for the best global fit at both stages, but the only significant change in the PDFs is for the u_V distribution for $x < 0.03$ at high $Q^2 \sim 10000 \text{ GeV}^2$, or slightly higher x at low Q^2 .

At present, and even in the foreseeable future, we will have to continue to include the existing

²The HERAPDF fit [5] uses fewer free parameters in their study. However, in that analysis the effect of adding extra parameters is included as part of the additional “parameterisation” uncertainty.

deep-inelastic scattering and Drell-Yan data from deuteron targets to achieve a determination of the PDFs of the different quark flavours, particularly at moderate and large values of x . It is therefore important to repeat our previous study of improving the nonperturbative corrections to the deuteron structure functions [10] from the default first used in [13], but now using the extended Chebyshev parametric forms of the input PDFs. The results are rather more successful than when using our standard PDF parameterisation, with the deuteron correction being rather similar to that expected from various models, with little variation when different assumptions are made. The change in deuteron corrections is found to change the d_V distribution to a fairly significant extent.

For the Chebyshev input parameterisations, without and with the additional freedom in deuteron corrections, we demonstrate that a suitable set of PDF uncertainty eigenvectors can be found, using 23 orthogonal directions in parameter space, rather than the 20 used in the standard MSTW2008 PDF set. Since it is the valence quark PDFs that are affected, we examine the detailed dependence of quark decomposition on the lepton charge asymmetry (that results from W^\pm production). We show that the precise combination, and the x range, of the PDFs probed is very dependent on the lepton rapidity and on the lepton p_T cut applied to the data.

The lepton charge asymmetry is particularly sensitive to the small- x valence PDFs, and this sensitivity is seen to increase rather dramatically as the p_T cut is raised. The predictions obtained from the new PDFs (using Chebyshev input parameterisations) are compared with both the Tevatron lepton asymmetry data [14] that were not used in the MSTW2008 fit, and the recent LHC data, namely the ATLAS lepton rapidity data [15] and the CMS lepton asymmetry data [16]. In all cases the default MSTW2008 PDFs are not optimum for these lepton asymmetry data sets, whereas the new ‘Chebyshev’ PDFs give much improved predictions even though they are obtained from a fit to *exactly* the same global data set as used to determine the MSTW2008 PDFs.

Although the lepton charge asymmetry, which has extreme sensitivity to the low x valence-sea decomposition, is better fit by the change in the PDFs, we check that the predicted values of the W^\pm , Z , Higgs, etc. total cross sections are essentially *unaltered*. To be precise, they are only changed by amounts far smaller than those due to the PDF uncertainty.

2 Parameterising Input PDFs with Chebyshev Polynomials

In a recent previous study [17], an investigation of the uncertainty of our PDFs using Monte Carlo generated data replicas was performed, as opposed to the use of perturbations about the best fit as was done in the MSTW2008 analysis. Little change was seen when the full 28 MSTW PDF parameters were left free compared to the 20 used in eigenvector generation. To be precise, the uncertainty using $\Delta\chi^2 = 1$ was compared using the two approaches and significant difference was only seen for the u_V distribution for $x < 0.03$ at the input scale $Q_0^2 = 1 \text{ GeV}^2$, where the uncertainty band expands a little, and for d_V in some x regions. It was particularly reassuring that there is little change in the uncertainty on the gluon distribution despite the number

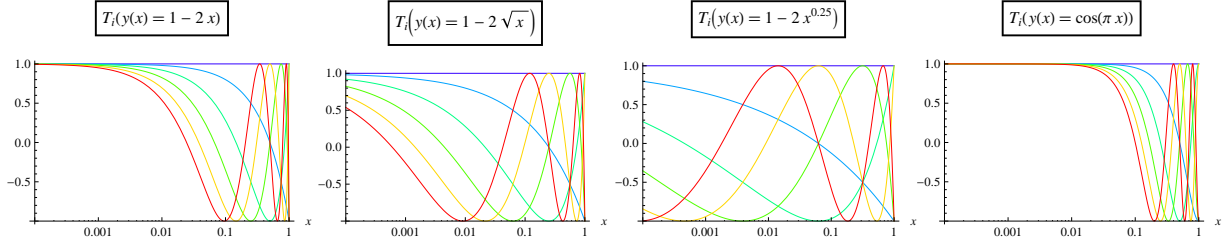


Figure 1: Behaviour of Chebyshev Polynomials $T_i[y(x)]$ of order $i = 0$ to 5 as a function of x for different arguments for the expansion variable. The order of the polynomial increases as the structure extends to smaller x values. The order of the polynomial also increases across the visible spectrum (i.e. dark blue to red).

of free parameters being extended from 4 to 7. Since it is difficult to apply our previously used ‘dynamical tolerance’ technique [1] for the uncertainty determination to this Monte Carlo method, and since there was little change in the results, it was concluded that the eigenvector approach was justified and would continue to be used in our PDF analyses³. Nevertheless, there was some evidence that an extended parameterisation might lead to some differences in the PDFs of the valence quarks. Hence, we start by investigating this hypothesis.

For valence and sea quarks the default MSTW parameterisation for the input at $Q_0^2 = 1 \text{ GeV}^2$ was taken to be

$$xf(x, Q_0^2) = A(1-x)^\eta x^\delta (1 + \epsilon x^{0.5} + \gamma x). \quad (1)$$

The $(1-x)$ power, η , allows a smooth interpolation to zero as $x \rightarrow 1$ and is inspired by number counting rules. The single small- x power, δ , is inspired by the behaviour predicted by Regge theory at small x . We found long ago that, first at NNLO [18], and also with improved data at NLO [19], that two terms with different small x powers were needed for the gluon distribution to give the best fit. For the gluon the parameterisation is

$$xg(x, Q_0^2) = A_g(1-x)^{\eta_g} x^{\delta_g} (1 + \epsilon_g x^{0.5} + \gamma_g x) + A_{g'}(1-x)^{\eta_{g'}} x^{\delta_{g'}}. \quad (2)$$

The input parameterisations for some other distributions, $\bar{d} - \bar{u}$ and $s - \bar{s}$, take slightly different forms, but these are not very precisely determined, and we will not consider changes to these in this article. Similarly, as previously, $s + \bar{s}$ is taken to be the same as the sea parameterisation except for the normalisation and $(1-x)$ power, which are left free. The polynomials, interpolating between the high- x and low- x limits, have no real motivation other than the separation of half-integer powers being again inspired by Regge theory, and the two free parameters seeming to be sufficient to obtain an optimum fit. An investigation of introducing either an extra parameter of the form ax^2 or $ax^{0.25}$ into the valence quark parameterisation was reported very briefly in [10] since neither had a significant effect on the fit quality – at best they gave $\Delta\chi^2 = -4$. However, the introduction of an ax^2 term did change the small- x u_V distribution a little outside

³It was, however, shown how an arbitrary number of Monte Carlo sets of PDFs could be generated starting with the eigenvector definition.

its uncertainty, and hence, as with the Monte Carlo study, suggests the uncertainty on this PDF, in the range $x < 0.03$, is underestimated.

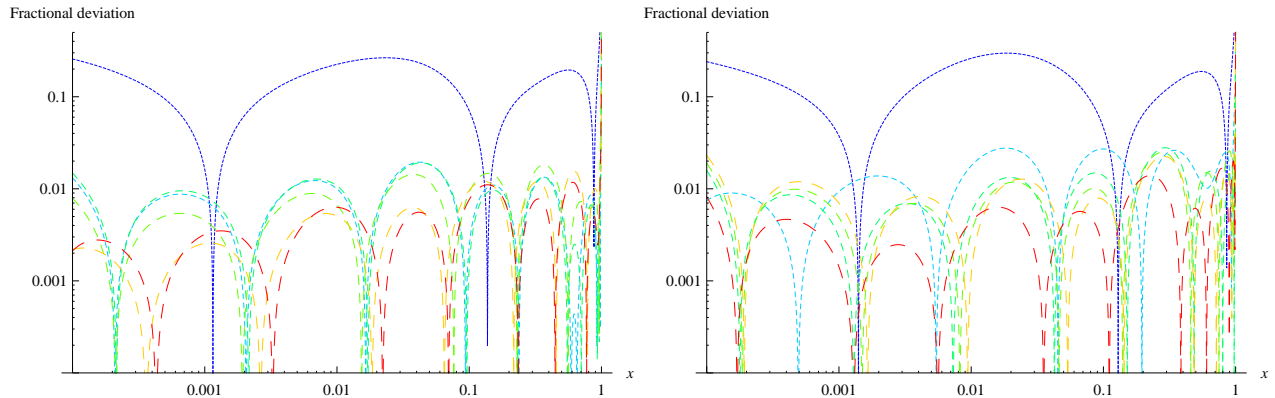


Figure 2: Two examples of the fractional deviation between fitted function and true function for fits with increasing highest order of Chebyshev Polynomials for valence-like distributions. The dash length decreases as the highest order of the polynomial increases. The order of the polynomial also increases across the visible spectrum (i.e. dark blue to red).

Here we undertake a much more systematic study. As a basis for the interpolating polynomial we decide to use Chebyshev polynomials (though we looked at, and will mention briefly, other possibilities). So we write

$$xf(x, Q_0^2) = A(1-x)^{\eta}x^{\delta} \left(1 + \sum_{i=1}^n a_i T_i(y(x)) \right), \quad (3)$$

where y is a function of x to be specified. We keep the same form of the $(1-x)$ and x powers in the high- and low- x limits. One of the main motivations for the choice of Chebyshev polynomials is that, not only the end points of the polynomials at $y = \pm 1$ have magnitude 1, but each maximum and minimum between the endpoints does also. Other choices, such as Legendre polynomials have maxima and minima with smaller magnitudes so they have smaller variations in magnitude away from the endpoints. There is still a choice to make regarding the argument y of the polynomial. We need $y = 1$ at the lower limit of x , i.e. $x = 0$ and $y = -1$ at the other limit $x = 1$, but there are many choices which could satisfy this. In practice the PDFs are measured between a range of roughly $0.0001 < x < 1$, so we want a choice such that the polynomials vary throughout the whole of this range. The form of the first few polynomials is shown for various choices in Fig. 1. Clearly $y = 1 - 2x$ is too concentrated at high x and $y = 1 - 2x^{0.25}$ extends to too low x . An alternative of $y = \cos(\pi x)$ is very concentrated at high x . We choose $y = 1 - 2\sqrt{x}$ as a convenient definition. This is the same choice as in the study reported in [7]. It is slightly different to the choice in [9] which used logarithmic dependence rather than powers of x , but the results are similar. A polynomial in $y \equiv 1 - 2\sqrt{x}$ also has the feature that it is equivalent to a polynomial in \sqrt{x} , the same as the default MSTW parameterisation, though for a n^{th} -order Chebyshev polynomial the maximum power of x is $x^{n/2}$. The half integer separation of terms is consistent with the Regge physics motivation of the MSTW parameterisation.

Pumplin [7] has explained clearly why a parameterisation like (3) is advantageous. Most previous parameterisations, including MSTW, have been based on interpolating functions like those in (1) with a small number of parameters, ϵ, γ, \dots . If the number of parameters is increased to allow more flexibility, the resulting fit becomes unstable, with parameters taking large values and with strong cancellations between the corresponding terms. On the other hand, the parameters, a_i , in the Chebyshev form (3) are much more convenient, and well-behaved, for fitting to the data. The requirement of smoothness in the input PDFs forces the values of the parameters a_i to be reasonably small at large order i . The Chebyshev polynomials of increasingly large order, n , model the behaviour of the input distribution at an increasingly fine scale in x . However, it is still an open question as to how many parameters are needed to model a parton distribution to sufficient accuracy without also starting to fit fluctuations. So far the standard technique is to impose some artificial restriction such as a requirement on smoothness of the function.

In order to test how many parameters are indeed needed for a sufficiently good fit we generate pseudo-data for a valence quark input PDF, say u_V , scattered around a function with the general shape of a valence quark distribution obtained from a very large order polynomial $f(x)$ with smoothness constraints applied in order to stop it developing kinks. The function is constrained to give an integrated total of two valence quarks and the fits are all constrained in the same manner. The 1000 pseudo-data points are distributed evenly in $\ln(1/x)$ with their percentage error held constant at 3%, i.e. $\text{error}_i = 0.03f(x_i)$. We then find the best fit for this pseudo-data using a parameterisation of the form (3) with increasing highest order of the Chebyshev polynomials. This procedure was repeated for a variety of different choices of starting function. Two typical results are shown in Fig. 2 which shows the percent deviation of the fit function from the full function for two different starting functions. The highest order of the polynomial increases across the visible spectrum (i.e. dark blue to red). Just using one term in the polynomial, i.e. $1 + a_1 T_1(y)$, can give deviations of about 10% over a wide range, but with 2 terms in polynomial this reduces to mainly a $\leq 2\%$ deviation (in most cases – sometimes 2 polynomials can still give significantly larger deviations $\sim 5 - 10\%$). For 4 terms there is generally $\leq 1\%$ deviation except at very high x . This does not improve very significantly with further terms added. This accuracy should be compared to the uncertainty in the MSTW2008 input PDFs. For valence quarks the 1 sigma uncertainty is at best just lower than 2% for $u_V(x, Q_0^2)$ near $x = 0.1$. For $d_V(x, Q_0^2)$ it is nearly always $> 3\%$.

In Fig. 3 we see the χ^2 distribution for increasing highest order of Chebyshev polynomials. This reflects the deviation from the original function, i.e. for one term in the polynomial there are many points with high χ^2 but the distribution becomes roughly as expected even with two just terms. There is no obvious structure as a function of x in the figure and extremely few χ^2 values, if any, take values greater than 10. The total χ^2 improves dramatically when going from 1 to 2 terms in the polynomial. After this it decreases by a few units with each additional polynomial up to 6th order, though this is difficult to appreciate from the plots. In some cases after n terms in the polynomial we start fitting noise, i.e. the χ^2 becomes lower than that for the true function. The number of polynomials required for this varies, but in the most extreme case happened with just 2. Somewhere between 4 – 6 is more common, but this feature is not always present when using 6 terms. Fits were also performed using Legendre polynomials. Since an term including up to the n^{th} order Legendre polynomial is just a re-expression of a term

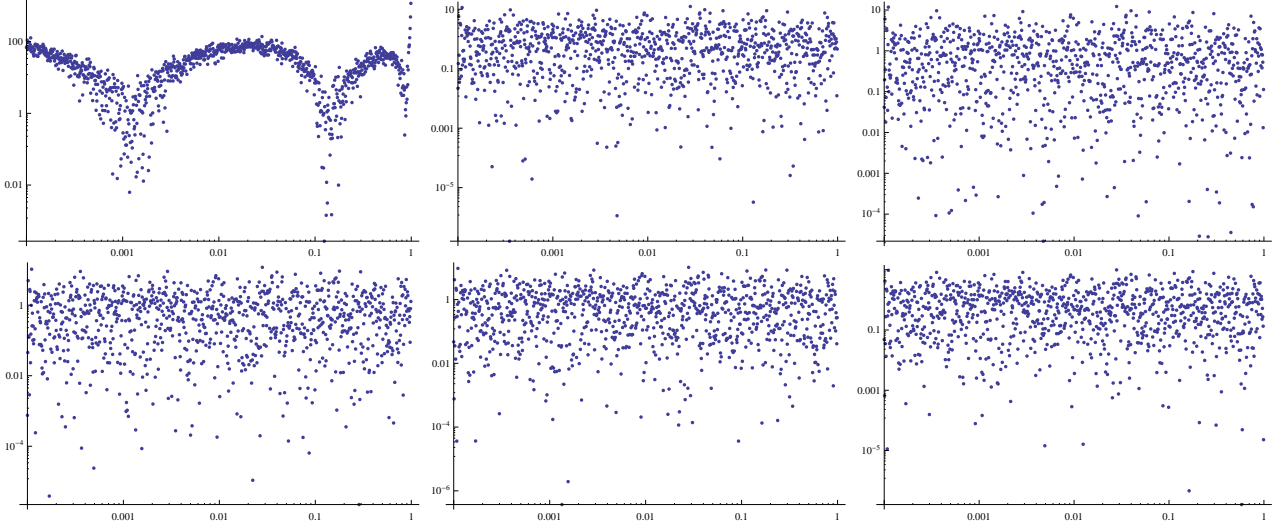


Figure 3: Distribution in χ^2 values for fits to a valence-like quark distribution with increasing highest order of Chebyshev Polynomials, going from 1st (top left) from left to right for the first row, then from left to right for the second row until the 6th order. Note that the vertical axis is not the same in all plots as the number of points with very large χ^2 decreases with the highest order of the polynomial. In this example, for two terms onwards there is a fairly random distribution.

including up to the n^{th} -order Chebyshev polynomial, the fit quality is the same within numerical accuracy. However, the coefficients are more correlated in the case of the Legendre polynomials, presumably because each term has less variation with x in this case. We have also checked that variation of the size of the error on the pseudo-data leads to no significant difference in the results, other than the values of the χ^2 , which becomes higher for poor fits as the error decreases and *vice versa*. For 3% uncertainty once the typical deviation is within a percent or so the χ^2 becomes close to one per point.

The exercise is repeated for a sea quark type distribution, i.e. falling more quickly at high x with increasing x , and growing far more quickly at small x with decreasing x , and for which there is no strong sum rule constraint. For this type of function the convergence to a very good fit is a little slower. Again one term in the polynomial gives a very poor fit and deviations $\sim 10\%$. This time addition of another term reduces the deviation to 3 – 4% at most and 4 polynomials usually results in deviation $\leq 2\%$ (except at high x). For deviations largely guaranteed to be $< 1\%$ 6 terms in the polynomial are usually required. The results for two examples are shown in Fig. 4. The greater difficulty in obtaining the excellent description in this case is presumably due both to the lack of the sum rule constraint in the function and the much wider variation in values for the PDF than the valence quark case. However, we should note that in this case of the sea quark distribution (or gluon distribution) the minimum 1 sigma uncertainty in the MSTW2008 input PDFs is $\sim 5\%$. Hence, the deviations with 4 parameters are again much smaller than the intrinsic uncertainty in the function and 4 parameters is very likely to be more than sufficient. In Fig. 5 we see the χ^2 distribution for increasing highest order of Chebyshev

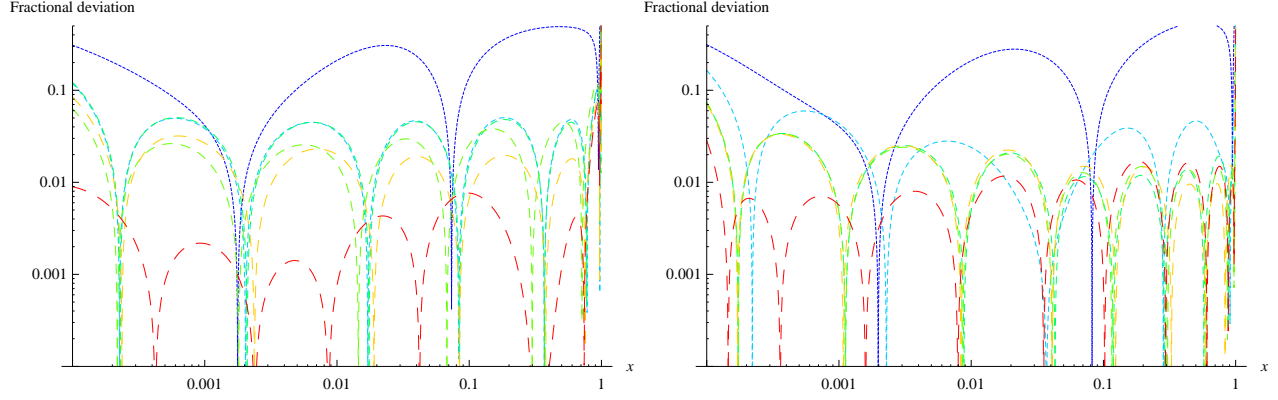


Figure 4: Two examples of the fractional deviation between fitted function and true function for fits with increasing highest order of Chebyshev Polynomials for sea-like distributions. The dash length decreases as the highest order of the polynomial increases. The order of the polynomial also increases across the visible spectrum (i.e. dark blue to red).

polynomials. Again we see that the distribution becomes essentially random with 3-4 terms in the polynomial, but it happens a little more slowly than in the case of a valence-like distribution. The total χ^2 values suggest that there is little evidence for over-fitting of the sea distribution until we get to terms with at least 6 Chebyshev polynomial.

We also investigate the case of fitting pseudo-data for a valence-like distribution, but where the generated pseudo-data are generated only for $0.01 < x < 0.68$, rather than for all x down to $x = 10^{-4}$. This is a more realistic situation, since valence quarks are only constrained by data in roughly this region. In this case two terms in the polynomial often give quite a very poor fit in some x regions within the fit range with deviations $\sim 10\%$. The addition of another term reduces the deviation to 4 – 5% at most and 4 polynomials usually results in deviation $\leq 1\%$, except at very high x and much lower x than the range of data. The results for two examples are shown in Fig. 6. The addition of more than 4 terms improves the comparison to the true function for the range of x containing data, but can increase the deviation in the very small- x regime. It can also often lead to over-fitting of the data points, which presumably contributes to the variation in the very low- x range. The χ^2 distribution as a function of x is shown in Fig. 7. As in the other examples, a uniform distribution sets in when there are 4 terms in the polynomial. Hence, again four terms seems sufficient, but there is more evidence that more than four terms corresponds to over-fitting in this case.

Finally we try fitting to the same type of pseudo-data, i.e. for a valence-like distribution with points $0.01 < x < 0.68$, but with only 100 pseudo-data points rather than 1000. The results are shown in Fig. 8. In this case the convergence is quicker, with deviation from the true function of 1% or less in the range of x where points exist with only three terms in the polynomial. In this case there is then no further improvement with extra parameters. For 100 points the match to the true function is already as good as the representation of the true function by 100 data points allows.

Overall we see that there is some dependence of the quality of the fit using a given number

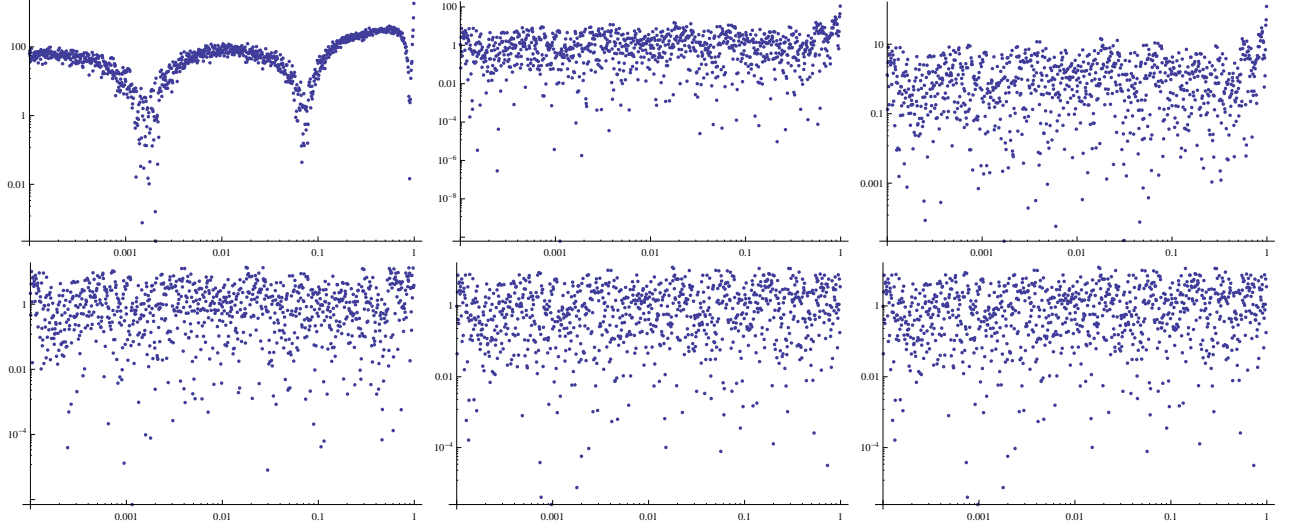


Figure 5: Distribution in χ^2 values for fits to a sea quark type of distribution with increasing highest order of Chebyshev Polynomials, going from 1st (top left) from left to right for the first row, then from left to right for the second row until the 6th order. Note that the vertical axis is not the same in all plots as the number of points with very large χ^2 decreases with the highest order of the polynomial. In this example, for four terms onwards there is a fairly random distribution. There is distinct structure for one term, and for two and even three terms a cluster of badly fit points at high x .

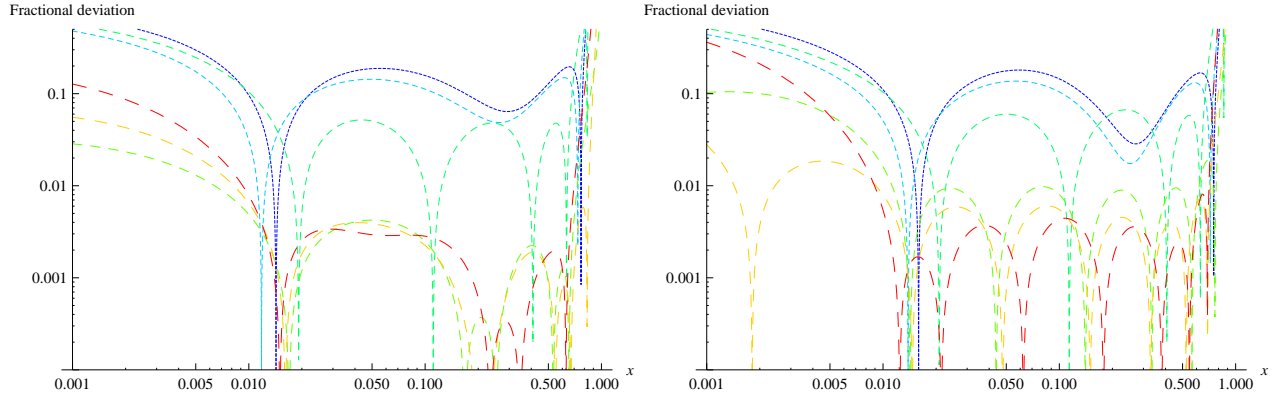


Figure 6: Two examples of the fractional deviation between fitted function and true function for fits with increasing highest order of Chebyshev Polynomials for valence-like distributions with 1000 pseudo-data between $0.01 < x < 0.68$. The dash length decreases as the highest order of the polynomial increases. The order of the polynomial also increases across the visible spectrum (i.e. dark blue to red).

of terms in the Chebyshev polynomial on shape of the PDF, whether there is a constraint on the function being fit, the x range of the data representing the function and the number of data points. However, it always seems to be the case that 4 terms are sufficient to get an accuracy

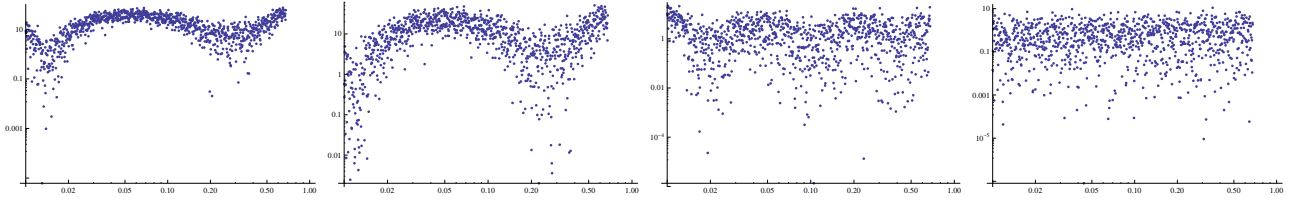


Figure 7: Distribution in χ^2 values for fits to a valence quark type of distribution with 1000 pseudo-data between $0.01 < x < 0.68$ for increasing highest order of Chebyshev Polynomials, going from 1st to 4th order from left to right. Note that the vertical axis is not the same in all plots as the number of points with very large χ^2 decreases with the highest order of the polynomial. In this example, there is structure for one, two and to some extent three terms, and only with four terms is there a fairly random distribution of χ^2 values.

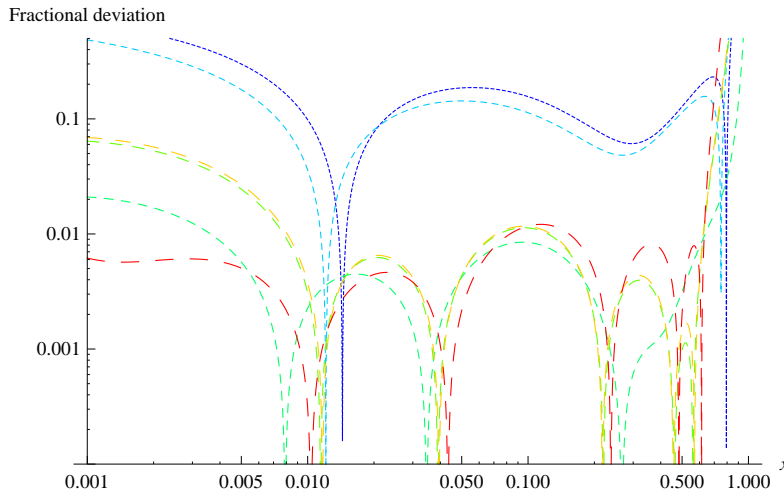


Figure 8: The deviation between fitted function and true function for fits with increasing highest order of Chebyshev Polynomials for a valence-like distribution with 100 pseudo-data between $0.01 < x < 0.68$. The dash length decreases as the highest order of the polynomial increases. The order of the polynomial also increases across the visible spectrum (i.e. dark blue to red).

considerably better than the uncertainty on the MSTW2008 input distributions. It also seems to be the case that many more than 4 terms leads to a distinct danger of over-fitting. Also, if there are relatively few data points then it is impossible to get a very accurate representation of the true function, and again too many terms in the polynomial can lead to over-fitting and instability in extrapolation to ranges of x outside the data constraint. This is relevant for PDFs with relatively weak data constraints, such as $s - \bar{s}$, $\bar{d} - \bar{u}$ and possibly $s + \bar{s}$.

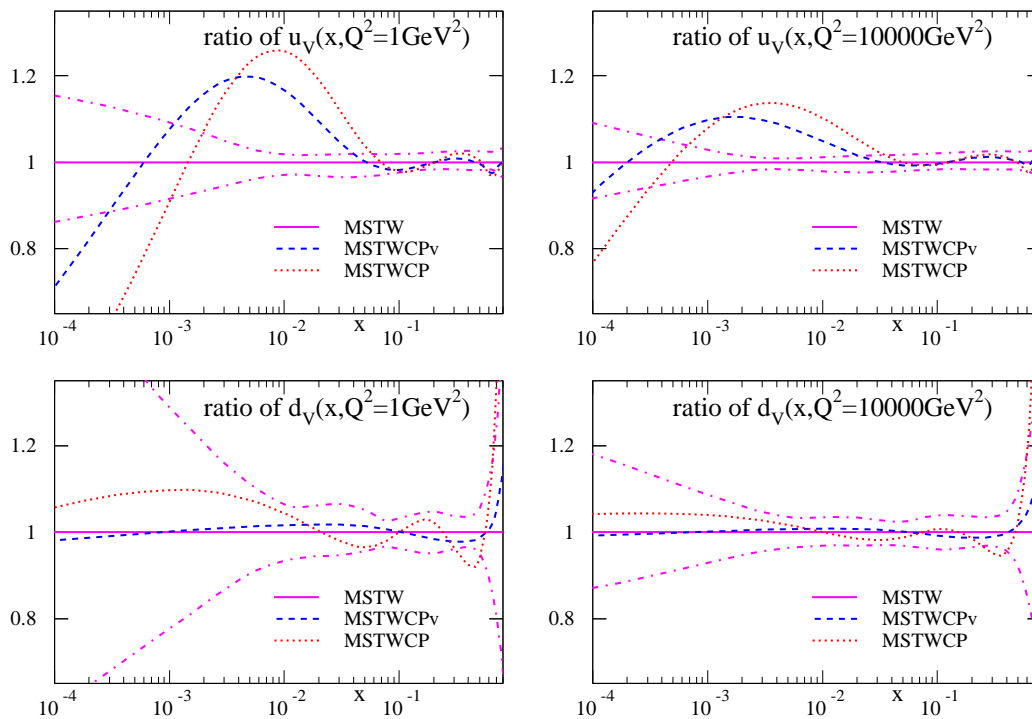


Figure 9: The change in the valence PDFs extracted from the MSTW2008 type fit using Chebyshev Polynomials for the valence quarks only (MSTW2008CPv) and for valence and sea quarks (MSTW2008CP) compared to the original MSTW2008 PDFs at NLO with their 68% uncertainties given by the dot-dashed lines.

3 Impact of Extended Parameterisations on PDF fits

Having studied the effects of fitting different ‘extended’ input PDF parameterisations to pseudo-data, we now investigate their effect in the real case of fitting to experimental data. The experimental data points are scattered over a wide range of Q^2 values, so both the evolution and the input distributions are required to be correct. Also, the data points are for structure functions, and other related high-energy scattering data, which, in general, not only depend on complicated combinations of PDF flavours, but are also related to them via the convolution with perturbative coefficient functions for the specific process. We perform the fits at next-to-leading order (NLO) in QCD perturbation theory, though MSTW also produce PDFs at leading order (LO) and next-to-next-to leading order⁴ (NNLO), and we will discuss NNLO results, which are very similar, later. Here, we perform fits to exactly the same data as used for the MSTW2008 PDF analysis [1], and adopt all the same theory decisions, e.g. heavy flavour schemes, nuclear target corrections, etc., but now with extended input PDF parameterisations.

To begin, we apply an extended parameterisation with Chebyshev polynomials of highest order $n = 4$ only for the valence quark PDFs: u_V and d_V . The resulting improvement to the

⁴At NNLO it is necessary to make some approximations in modelling unknown coefficient functions for some processes.

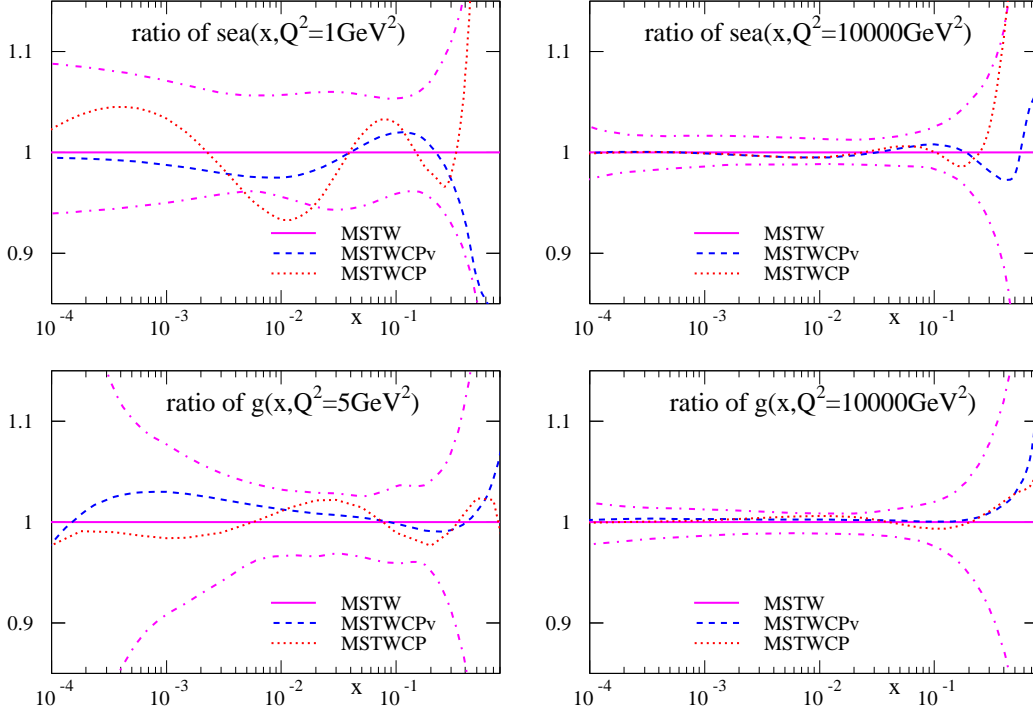


Figure 10: The change in the sea and gluon PDFs extracted from the MSTW2008 type fit using Chebyshev Polynomials for the valence quarks only (MSTW2008CPv) and for valence and sea quarks (MSTW2008CP) compared to the original MSTW2008 PDFs at NLO with their 68% uncertainties shown by the dot-dashed lines. The gluon is shown at $Q^2 = 5 \text{ GeV}^2$ rather than the input scale of $Q^2 = 1 \text{ GeV}^2$ as the fact it goes negative at small x at the latter Q^2 makes a ratio plot unclear.

global fit is quite minor; corresponding to $\Delta\chi^2 = -4$ compared to a total of 2543 for 2699 data points in the MSTW2008 fit. There is a large improvement in the description of the BCDMS structure function data [20], but a deterioration in the fit to NMC structure function data [21]. This is very similar to the results obtained previously by adding just an ax^2 term to the valence parameterisations [10], and, indeed, is similar since Chebyshev polynomials in $(1 - 2\sqrt{x})$ with highest order $n = 4$ do add an x^2 term, but also a $x^{1.5}$ term. As in this previous study, the significant change is in $u_V(x)$ for $x \leq 0.03$ at $Q^2 = 10000 \text{ GeV}^2$. However, it is a larger change than previously. The comparison to the MSTW2008 PDFs at $Q_0^2 = 1 \text{ GeV}^2$ and at $Q^2 = 10^4 \text{ GeV}^2$ with uncertainty bands is shown in Figs. 9 and 10.

A fit is also attempted using $y = (1 - 2x^{0.25})$ as the argument of the Chebyshev polynomials, which overlaps with using an extra $x^{0.25}$ term in the parameterisation as tried before. As in the previous study [10], this results in an improvement in the fit of less than one unit in χ^2 , and much less change in PDFs. Hence, the use of $y = (1 - 2\sqrt{x})$ receives further justification.

We then also applied the extended Chebyshev interpolating polynomial to the sea distribution. For the sea, the MSTW2008 parameterisation was exactly the same form as for the valence quark, i.e. as in (1), so the extended parameterisation also has the same form as that for the

valence quark PDFs. The only difference is that there is no number sum rule directly constraining one parameter, unlike the valence quark PDFs where the normalisation is constrained. We apply the extended parameterisation for the sea distribution by replacing $(1 + \epsilon x^{0.5} + \gamma x)$ by a term including Chebyshev polynomials with highest order $n = 4$. We tried also extending the parameterisation for the gluon distribution. For the MSTW2008 gluon distribution we had the more flexible parameterisation in (2) where, in practice, the second term chooses a negative normalisation. The normalisation of the first term is set by the momentum sum rule for the PDFs, so in practice there are 7 free parameters. We replaced the polynomial in the first term by one including Chebyshev polynomials with highest order $n = 4$, but both the quality of the fit and the resulting PDFs were essentially unchanged. This, together with the fact that the gluon already has 7 free parameters, suggests that the extended Chebyshev polynomial parameterisation is not necessary. However, using Chebyshev polynomials is a more efficient way of expressing the polynomial in the first term of (2), so we replaced the previous form by an entirely equivalent term using Chebyshev polynomials with highest order $n = 2$. The resulting global fit for the parameterisation with the extended sea quark distribution, and the formally modified, but equivalent, gluon distribution, has $\Delta\chi^2 = -29$. The improvement is mainly for BCDMS structure function data and E866 Drell-Yan [22] and Drell-Yan asymmetry data [23]. Again there is a deterioration in the description of the NMC structure function data, but this time a slight improvement in Tevatron lepton asymmetry [24, 25] and Z rapidity data [26, 27]. The PDFs are again shown in Figs. 9 and 10, and compared to the MSTW2008 PDFs and their uncertainty. The change in u_V is similar to the previous case. There is more change in d_V this time, but generally within the 68% uncertainty band. The sea quarks at input show some differences, sometimes a little outside the 68% confidence level uncertainty, but this is essentially washed out by evolution for $Q^2 = 10^4 \text{ GeV}^2$. All other PDFs show changes that are very small compared to the MSTW2008 uncertainty.

In all our new fits we let $\alpha_S(M_Z^2)$ be a free parameter and in all cases it changes by only 0.0002 or less, i.e. a change much smaller than the uncertainty of $+0.0012$ or -0.0015 at NLO [28]. The set of PDFs, with Chebyshev polynomials of highest order $n = 4$ applied to the two valence quarks and sea, and Chebyshev polynomials of highest order $n = 2$ to the gluon, is denoted by ‘MSTW2008CP’ below.

As well as studying the central values, we also investigate the uncertainties of the new PDFs. The standard MSTW2008 PDFs have 28 free parameters in the best fit, but 8 are held fixed when determining the uncertainty eigenvectors because there is too much correlation or anticorrelation between some of the parameters when all are left free. This freedom would have resulted in the 8 extra potential eigenvectors having an extremely non-quadratic behaviour in $\Delta\chi^2$; in general behaving quadratically only in the immediate vicinity of the minimum, then with $\Delta\chi^2$ increasing alarmingly away from this limit.

In the MSTW2008 analysis, all PDFs, except the sea (and $s - \bar{s}$ which only has two free parameters), have the small- x and high- x powers δ and η free in the eigenvector determination, the small- x power being replaced by the normalisation for the sea. For the gluon the 4 free parameters in the eigenvector determination are the two η and δ values in (2). For valence and sea quarks we then also let the coefficient of the $x^{0.5}$ term in the polynomial, ϵ , be free.

For the determination of the uncertainties of the MSTW2008CP PDFs, we decide to apply

more consistency between the PDFs, and for the sea quarks we let the parameter δ be free in the eigenvector determination, rather than the normalisation. For all the PDFs, other than valence quarks and the light sea, we make the same choices as usual. For the valence and sea quark PDFs we let the coefficients of the first and third Chebyshev polynomials, a_1 and a_3 , be free in the eigenvector determination. Hence we have one more free parameter for each of these PDFs and consequently 23 rather than 20 eigenvectors. Despite having one extra free parameter the uncertainty on sea quarks for $x \sim 0.001 - 0.01$ at $Q^2 = 10000 \text{ GeV}^2$ becomes a little smaller, but becomes noticeably larger for very low x . This is undoubtedly due to the exchange of the normalisation for the small- x power as a free parameter. The most significant change in the uncertainty, however, is in the same PDF as for the change in the central value, i.e. the small- x u_V distribution. The uncertainty starts to become larger with decreasing x at a higher value of x , i.e. about $x = 0.01$. Indeed, it is markedly larger between 0.001 and 0.01 at $Q^2 = 10000 \text{ GeV}^2$, where there was a rather artificial ‘neck’ in the MSTW2008 uncertainty of u_V near $x = 0.003$, and is significantly larger at very small x values. Hence, in the new MSTW2008CP analysis, the increase in uncertainty in u_V is more in line with where the data constraint finishes, i.e. about $x = 0.01$. All 23 eigenvectors have reasonably quadratic behaviour, the worst being comparable to the worst for MSTW2008. The worst is in fact the eigenvector comprising largely of the first term in the Chebyshev polynomial for d_V . This is explained by the fact that this parameter is highly correlated with both the small- x power and the third term in the Chebyshev polynomial for the same PDF.

4 Fitting PDFs with Deuteron Corrections Applied

At present, and for the foreseeable future, it is necessary to include the deep-inelastic scattering data on deuteron targets in the global parton analyses, in order to separate the u and d PDFs at moderate and large x values. Studies of the PDFs obtained with collider data only [6, 17] show much bigger uncertainties for some PDF flavours. Unfortunately, the deuteron measurements are subject to nuclear corrections. With the increased precision and variety of data (especially with the advent of the decay lepton charge asymmetry measurements from W^\pm production at the Tevatron and LHC) it is necessary to study the effect of the deuteron corrections in some detail. In previous PDF determinations we have only included a fixed shadowing correction for small x . In the PDF analysis of [3], a specific deuteron correction, with associated uncertainty, was applied. Most of the other groups include no deuteron corrections (the issue does not arise in [5]), although there have been recent specific investigations (see e.g. [29, 30] in the context of the CTEQ-JLab fits, although there are no corrections in [2]) which suggest the effect is not insignificant, especially at high x . As we will explain below, we base our study in this article on the assumption that there need not be deuteron corrections, but allow the fit to choose them if required, with some uncertainty determined by the fit quality. Hence, at the very least we allow some new degree of PDF uncertainty to be associated with the possibility of deuteron corrections. The best fit chooses some small, but significant, corrections at high x .

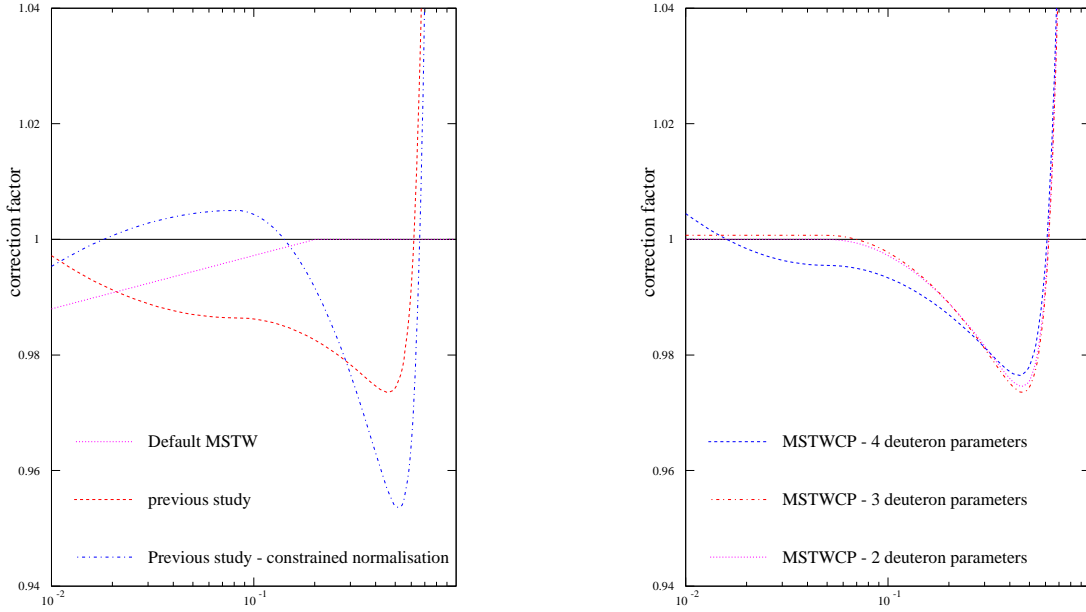


Figure 11: The deuteron corrections applied to structure functions in previous versions of the MSTW2008 type fit (left) and in fits where the extended parameterisation for the PDFs is used (right).

4.1 The parameterisation of the deuteron corrections

In the default MSTW2008 PDF analysis, deuteron structure functions were corrected only for shadowing at small values of x [12] with a negative correction starting a little above $x = 0.1$ and becoming as much as -1.5% near $x = 0.01$. In [10] we presented a much more detailed study. So, first, we briefly summarize the results of this investigation. Basically, we studied the effect of allowing the deuteron corrections to be described by forms with 4 free parameters, which were allowed to vary with no penalty. It was found that the quality of the global fit could improve by a very large amount with the optimum deuteron corrections. It was particularly clear that the comparison to the Tevatron lepton asymmetry data used in the fit improved a great deal, as did the predictions for more recent versions of these data. The deuteron corrections required were of the expected general form, see e.g. [29, 31, 32], with a large positive correction at very high x and a dip for $x \sim 0.5$. However, the dip was somewhat larger than expected, the correction remained negative near $x = 0.1$ where it is likely to be positive due to antishadowing effects, and if anything the fit preferred positive corrections near $x = 0.03$ rather than the negative shadowing corrections. (Indeed, if the shadowing corrections applied in the MSTW2008 analysis were to be simply removed, and no deuteron corrections applied, then the fit would improve slightly.) Hence, the adopted corrections seemed unsatisfactory.

Given that the extended ‘Chebyshev’ parameterisation discussed so far automatically allows improvement in the global fit, and has by far the most effect on valence quarks and the light sea, it seems natural to investigate the question of deuteron corrections in this context. The

deuteron corrections applied previously [10] were of the form

$$F^d(x, Q^2) = c(x)(F^p(x, Q^2) + F^n(x, Q^2))/2, \quad (4)$$

where $F^n(x, Q^2)$ is obtained from $F^p(x, Q^2)$ just by swapping up and down quarks, and anti-quarks, i.e. isospin symmetry is assumed. The correction factor $c(x)$ is taken to be Q^2 independent for simplicity and is of the form

$$c(x) = (1 + 0.01N_c)(1 + 0.01c_1 \ln^2(x_p/x)), \quad x < x_p, \quad (5)$$

$$c(x) = (1 + 0.01N_c)(1 + 0.01c_2 \ln^2(x/x_p) + 0.01c_3 \ln^{20}(x/x_p)), \quad x > x_p. \quad (6)$$

x_p is a “pivot point” for which value the normalisation is set to be $(1 + 0.01N_c)$. For $x < x_p$ there is the freedom to increase or decrease smoothly. The same is true above $x = x_p$, but the very large power is also added to allow the expected rapid change of the correction as $x \rightarrow 1$ due to Fermi motion. In previous studies x_p was chosen to be 0.08 but here we set $x_p = 0.05$. If there is shadowing at low x and also a dip for high, but not too high, x then x_p is where the correction would take its maximum value, expected to be determined by antishadowing corrections. Thus the 4 free parameters describing the deuteron correction, $c(x)$, are the c_i and N_c . We do not apply the corrections to the E866 data on Drell-Yan asymmetry [23], and this could be improved in future. However, in the region of the majority of (and most precise) data the correction is very small. Very naively the unconstrained deuteron correction can simply allow the deuteron structure function data to be fit as well as possible while other data sensitive to the separation between up and down quarks determine the PDFs. However, there are other constraints, such as sum rules, and in practice the many different types of structure function and other data, all depending on different combinations of flavours, in a global fit, makes the situation more complicated. In principle, extremely precise collider data will make the fit to deuteron data a more-or-less direct fit of deuteron corrections, but this is not yet the case with present data.

The deuteron correction [12] for the default MSTW2008 fit is shown in the left of Fig. 11. It is negative, i.e. the total correction factor is < 1 below about $x = 0.2$, but becomes larger in magnitude as x decreases. The correction factor for the best fit in our previous study [10] is also shown. As explained, it is negative everywhere, except at very high x , which seems unlikely. This gives an improvement in χ^2 compared to our usual global fit of ~ 90 . If the normalisation at x_p was fixed to be 1.005 the correction factor obtained had the expected type of shape, i.e. turned below 1 at the lowest x and dipped to a minimum near $x = 0.6$. However, this resulted in a fit with χ^2 30 higher than the free deuteron correction, and as seen the dip is now -5% , which is much lower than shadowing models tend to predict. The shape is very different to the correction with all parameters left free. Fixing the normalisation to 1.0025 and setting c_2 so that the dip is more like -3% , results in a further deterioration of $\Delta\chi^2 = 5$ to the fit quality. This is not particularly significant, but there seemed to be a tension between the best fit and the expected shape of the correction.

4.2 Deuteron corrections for ‘Chebyshev’ parameterisations

We now repeat the exercise using our extended ‘Chebyshev’ parameterisation for the valence quarks and sea distribution. The quality of the fit is nearly 100 units of χ^2 better than that for

the MSTW2008 fit, i.e. a little better than when we used the standard PDF parameterisation in our previous study [10]. There are very large improvements in the fit to (i) the BCDMS deuteron structure function data (with no corresponding deterioration in the description of the NMC data), (ii) the E866 Drell-Yan asymmetry data and (iii) the Tevatron lepton asymmetry data. The last improves from $\chi^2/N_{\text{pts.}} = 55/32$ in the MSTW2008 fit to $\chi^2/N_{\text{pts.}} = 27/32$. The deuteron correction is shown in the right of Fig. 11. As with our previous study, when all parameters are left free, the normalisation is smaller than 1, but this time by only 0.5%, rather than over 1.5%. There is also still a tendency for the correction to turn up, rather than down, at lowest x . Since this last feature is unexpected, we investigate it by fixing $c_1 = 0$, so there is no turn up, but no turn down either. This only changes the fit quality by 2–3 units (only a couple of data points really preferring the turn-up), and the normalisation is now 1.007. We also try setting the normalisation exactly to unity; the fit quality and deuteron correction are almost unchanged. In all three of these fits, the dip minimises at a value of about 0.975 at $x = 0.5$. Hence, the deuteron correction is stable as the number of parameters held fixed changes, as is the quality of the fit. Moreover, with the exception of the slight tendency to prefer an upturn near $x = 0.01$, the shape is as predicted by standard models [29, 31, 32].

4.3 The ‘MSTW2008CPdeut’ fit

Bearing this in mind, we take the fit with $c_1 = 0$ as the preferred fit since, while it produces no small x shadowing, it produces no enhancement either. It may be that the expected shadowing is not significant in practice until x below 0.01. Also, it may be expected that the value in the region $x = 0.05$ is larger than the 1.007 found, but this is far from certain. Using the PDFs from this fit, which we call ‘MSTW2008CPdeut’, we find that the prediction for the higher luminosity DØ lepton asymmetry data [14] integrated over all p_T greater than 25 GeV gives a fit of quality $\chi^2/N_{\text{pts.}} = 28/12$. Although this number seems high, this is close to the best that seems possible given the fluctuations in these measurements, and is close to the best that we have obtained even when fitting these data with a very high weight. We do not get a good fit to the data split into two different p_T bins, but this seems to be a problem found by other PDF fitting groups [2, 33].

The valence quarks resulting from the fit with deuteron corrections (MSTW2008CPdeut) are shown compared to the corresponding MSTW2008 PDFs in Fig. 12. For all PDFs, other than valence quarks, there is negligible change compared to those with just the extended parameterisation (MSTW2008CP), i.e. only the sea distribution changes at all significantly compared to MSTW2008 and in a similar way. The u_V distribution also shows little further change. All the significance is in the change of d_V . This increases by a little more than the 68% confidence level uncertainty at most near $x = 0.6$ and decreases by the 90% confidence level uncertainty at x near 0.03, the precise value of x decreasing with increasing Q^2 . This is as one might expect, since the deuteron correction is now larger than the MSTW2008 default at low x , so the down quark can be smaller, whereas the major increase at high x is in the same position as the minimum of the dip in the deuteron correction.

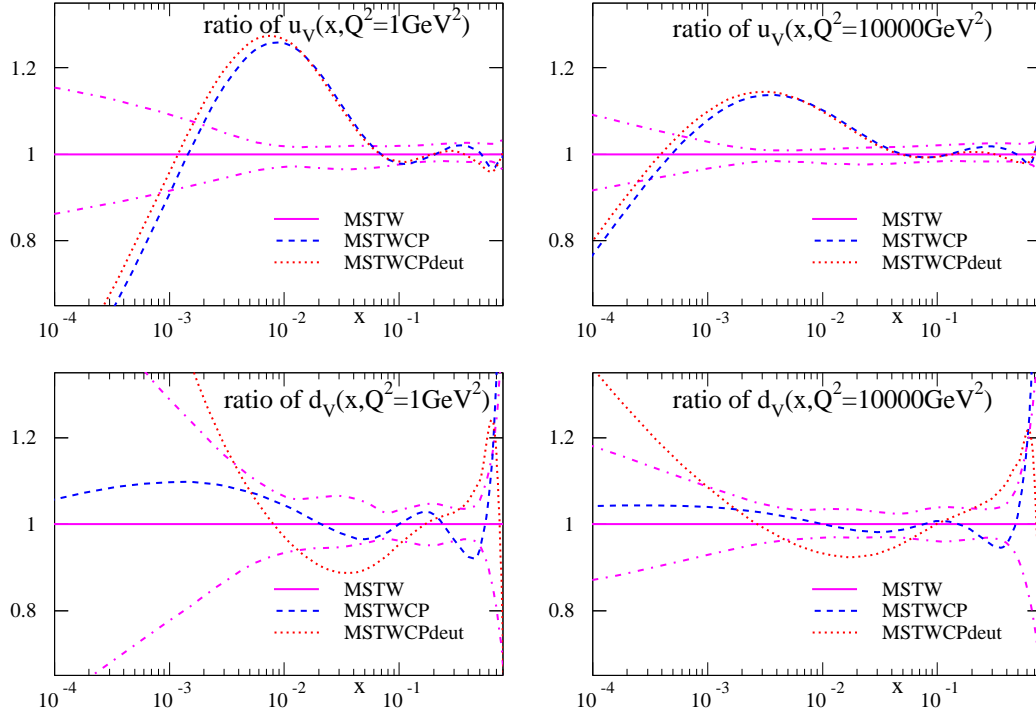


Figure 12: The change in the valence quark PDFs extracted from the MSTW2008 type fit using Chebyshev Polynomials and deuteron corrections (MSTW2008CPdeut) compared to the original MSTW2008 PDFs at NLO with their 68% uncertainties shown using dot-dashed lines.

4.4 Allowing for uncertainties on deuteron corrections

We also generate uncertainty eigenvector sets whilst applying deuteron corrections. Doing this with the deuteron corrections fixed at the position of the best fit would be straightforward, but would not account for the uncertainty in the deuteron corrections themselves. Since our best fit is of roughly the form one would expect for these corrections, and since there is no solid basis on which to judge quite how much variation in deuteron corrections is allowed, we choose to simply let the parameters in the deuteron correction go free with no penalty. This is then very similar to our procedure for heavy nuclear corrections, necessary for including neutrino deep-inelastic scattering data in the MSTW2008 analysis, where we take a set of corrections obtained from a global fit to nuclear data [34], but multiply by a function, similar in form to (6), which allows variations away from the default form with no penalty. In that case, in practice, the variations are small, i.e. our fit is very compatible with the determined nuclear corrections and the uncertainty in the nuclear corrections determined by the fit quality is a few percent, which seems entirely reasonable. Here we are doing exactly the same thing except that we have no starting deuteron correction, other than implicitly zero correction, to act as a template. Since deuteron corrections are expected to be small, and some groups use zero correction (as have we as default, i.e. in the MSTW2008 fit and previously, at high x), using no correction as the template seems reasonable. There is, however, a complication. There are 4 free parameters in our deuteron correction whereas we have 3 for our nuclear correction function,

and the deuteron structure functions are closely related to the down quark distribution. Hence, there are strong correlations between the 4 deuteron correction parameters, and between them and the parameters for d_V . It is impossible to get a stable perturbation about the best fit in terms of eigenvectors letting all 4 deuteron parameters go free. However, we did not even let all parameters go free in the best fit, choosing to fix c_1 in order to avoid an unlikely tendency for the correction to grow at the smallest x values. Letting it go free when determining eigenvectors would be inconsistent. Hence we let N_c, c_2 and c_3 go free when obtaining the eigenvectors. The freedom in the normalisation lets the correction at low x vary and there is not much distance between the lowest x data points and the pivot point at which the normalisation is set.

We attempt to construct 23 eigenvectors by letting the same parameters as before go free. However, this results in some severely non-quadratic behaviour. The worst case is the eigenvector comprising largely of the first term in the Chebyshev polynomial for d_V . As for the fit without deuteron corrections this parameter is highly correlated with both the small- x power and the third term in the Chebyshev polynomial for the same PDF, but the freedom in deuteron corrections make this correlation, and its effects, worse. This is no longer an acceptable eigenvector. Even fixing one more of the deuteron parameters does not help very strongly. Since the main problem is the correlation between the first Chebyshev polynomial and the small- x power, and higher-order polynomials are less influential at small x , we try instead letting the second and third Chebyshev polynomials have the free parameters for the valence quarks and sea when finding eigenvectors. This does indeed reduce the correlation between the parameters for d_V . It increases the correlation between the parameters for u_V , but this does not seem to translate into particularly bad behaviour of the eigenvectors, and this change provides 23 orthogonal eigenvectors with none having worse non-quadratic behaviour than any of the 20 in the MSTW 2008 fit. Hence, we have a preliminary set of uncertainty eigenvectors incorporating both the extended ‘Chebyshev’ parameterisation and the uncertainties due to deuteron corrections.

When the fit with deuteron corrections (MSTW2008CPdeut) is compared to the fit using just the extended parameterisation (MSTW2008CP), the u_V uncertainty increases a little bit more quickly below $x = 0.05$. However the effect for d_V is more dramatic – it is more than twice as uncertain as that for MSTW2008 for $x \sim 0.4$, and also about twice as uncertain for $x = 0.01 - 0.05$, but not near $x = 0.1$ where the data constraints are strongest. This expanded uncertainty means that the MSTW2008 d_V distribution is either within, or just outside the one-sigma uncertainty band for d_V obtained in the fit with deuteron corrections, except for $x < 0.0005$. For $u_V - d_V$ the uncertainty is more than twice as big as MSTW2008 for $x \sim 0.4$ and 30 – 50% bigger for $x = 0.05 - 0.01$, but only slightly larger near $x = 0.1$. The change in the central value and uncertainty for $u_V - d_V$ has important implications for the description of the lepton charge asymmetry, as we shall now see.

5 Lepton Asymmetry at the LHC and PDF Sensitivity

The measurements of the lepton charge asymmetry, from $W^\pm \rightarrow \ell^\pm \nu$ production and decay, at the Tevatron and the LHC, probe novel combinations of PDFs. In the next Section we shall investigate the effects of the extended ‘Chebyshev’ parameterisation, and of the deuteron

corrections, on the MSTW predictions for the observed asymmetries. However, first, to gain insight into the use of these data in PDF analyses, we explore the predicted behaviour of the lepton charge asymmetry at the LHC, based on the LO and zero width expressions for W production and decay, using MSTW2008 NLO PDFs. The NLO and NNLO corrections [35, 36] do not change the general picture significantly, though do change the precise values. In particular, we explain how the PDFs result in the interesting dependence of the asymmetry, shown in Fig. 13, on the experimental minimum p_T cut applied to the transverse momentum of the decay lepton.

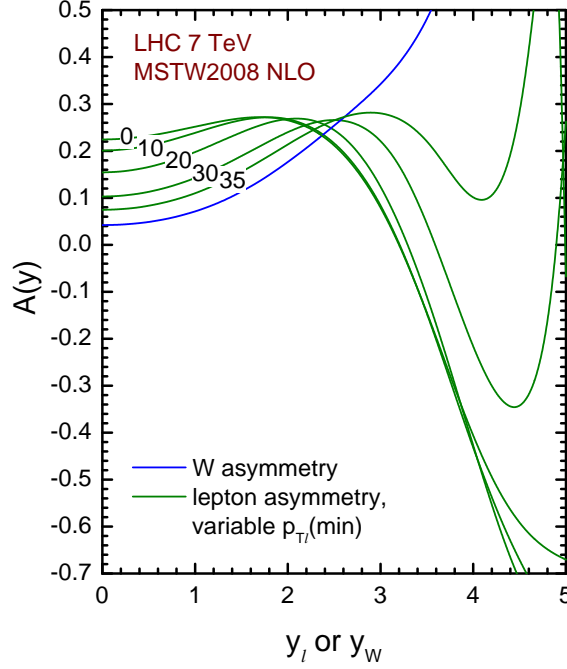


Figure 13: The dependence of the asymmetry on the lepton minimum p_T cut. The asymmetry is calculated at leading order and zero width using MSTW2008NLO PDFs.

We begin by considering the W charge asymmetry, defined by

$$A_W(y_W) = \frac{d\sigma(W^+)/dy_W - d\sigma(W^-)/dy_W}{d\sigma(W^+)/dy_W + d\sigma(W^-)/dy_W}, \quad (7)$$

where y_W is the rapidity of the W boson. At leading order, assuming u , d quark and antiquark contributions only, taking the CKM matrix to be diagonal, and writing $u = u_V + \bar{q}$, $d = d_V + \bar{q}$, $\bar{u} = \bar{d} \equiv \bar{q}$ (which is an approximation, which becomes more accurate at very small x), we have

$$A_W(y_W) \approx \frac{u_V(x_1)\bar{q}(x_2) + \bar{q}(x_1)u_V(x_2) - d_V(x_1)\bar{q}(x_2) - \bar{q}(x_1)d_V(x_2)}{u_V(x_1)\bar{q}(x_2) + \bar{q}(x_1)u_V(x_2) + d_V(x_1)\bar{q}(x_2) + \bar{q}(x_1)d_V(x_2) + 4\bar{q}(x_1)\bar{q}(x_2)}, \quad (8)$$

where $x_{1,2} = (M_W/\sqrt{s})\exp(\pm y_W)$. Contributions from c, s quark scattering can be approximately taken into account by $4\bar{q}(x_1)\bar{q}(x_2) \rightarrow 4(1 + \delta)\bar{q}(x_1)\bar{q}(x_2)$ in the denominator. Two

important limits are $y_W = 0$ for which $x_1 = x_2 = x_0 \equiv M_W/\sqrt{s}$, and $y_W \rightarrow y_W^{\max} = -\log(x_0)$ for which $x_1 \rightarrow 1$ and $x_2 \rightarrow x_0^2$. Thus

$$A_W(0) \approx \frac{u_V(x_0) - d_V(x_0)}{u_V(x_0) + d_V(x_0) + 2\bar{q}(x_0)} > 0, \quad A_W(y_W^{\max}) = 1. \quad (9)$$

In practice, it is usually the lepton charge asymmetry which is measured, defined in a similar way as

$$A(y_\ell) = \frac{d\sigma(\ell^+)/dy_\ell - d\sigma(\ell^-)/dy_\ell}{d\sigma(\ell^+)/dy_\ell + d\sigma(\ell^-)/dy_\ell}, \quad (10)$$

where y_ℓ is the (pseudo)rapidity of the charged lepton.⁵ Defining θ^* to be the emission angle of the charged lepton relative to the proton beam with positive longitudinal momentum in the W rest frame, then $\cos^2 \theta^* = 1 - 4p_T^2/M_W^2$, where p_T is the lepton transverse momentum. The rapidities are related by

$$y_\ell = y_W + y^*, \quad y^* = \frac{1}{2} \ln \left(\frac{1 + \cos \theta^*}{1 - \cos \theta^*} \right). \quad (11)$$

The leading-order parton momentum fractions are then

$$x_{1,2} = x_0 \exp(\pm y_W) = x_0 \exp(\pm y_\ell) \kappa^{\pm 1}, \quad \kappa = \left(\frac{1 + |\cos \theta^*|}{1 - |\cos \theta^*|} \right)^{1/2} > 1, \quad (12)$$

i.e. for a given p_T in $0 \leq p_T \leq M_W/2$, there are two solutions corresponding to positive or negative $\cos \theta^*$, or equivalently positive or negative y^* .

Neglecting overall factors, the analogue of the numerator of (8) for the lepton asymmetry (10) can be approximated by

$$\begin{aligned} & (u_V(x_1^+) \bar{q}(x_2^+) - \bar{q}(x_1^+) d_V(x_2^+) + u_V(x_1^-) \bar{q}(x_2^-) - \bar{q}(x_1^-) d_V(x_2^-)) (1 - \cos \theta^*)^2 \\ & + (\bar{q}(x_1^+) u_V(x_2^+) - d_V(x_1^+) \bar{q}(x_2^+) + \bar{q}(x_1^-) u_V(x_2^-) - d_V(x_1^-) \bar{q}(x_2^-)) (1 + \cos \theta^*)^2, \end{aligned} \quad (13)$$

where

$$\begin{aligned} x_1^+ &= x_0 \exp(+y_\ell) \kappa > x_1^- = x_0 \exp(+y_\ell) \kappa^{-1} \\ x_2^+ &= x_0 \exp(-y_\ell) \kappa^{-1} < x_2^- = x_0 \exp(-y_\ell) \kappa. \end{aligned} \quad (14)$$

The explicit θ^* dependent terms in (13) originate in the $V \pm A$ structure of the W couplings to fermions. Table 1 lists the values of the various quantities that enter in the expression for the lepton asymmetry as functions of p_T .

Note that whether or not one or both x solutions are physical depends on the values of y_ℓ and κ (i.e. p_T). For the range of lepton p_T accessible to experiment at LHC, there will always be two solutions for sufficiently small y_ℓ . Then as y_ℓ increases, the ‘+’ solution disappears first for $x_1^+ > 1$, i.e. $y_\ell > -\log(x_0 \kappa)$, and both solutions disappear for $x_1^- > 1$, i.e. $y_\ell > -\log(x_0/\kappa)$. For example, for $p_T = 20$ GeV at $\sqrt{s} = 7$ TeV, these limiting values are $y_\ell = 3.14$ and 5.79

p_T (GeV)	$ \cos \theta^* $	$ y^* $	$(1 + \cos \theta^*)^2$	$(1 - \cos \theta^*)^2$	κ
0	1.000	∞	4.00	0.00000	∞
5	0.992	2.77	3.97	0.00006	16.0
10	0.969	2.07	3.88	0.00099	7.91
15	0.928	1.64	3.72	0.00522	5.17
20	0.867	1.32	3.49	0.01757	3.75
25	0.783	1.05	3.18	0.04705	2.87
30	0.666	0.80	2.77	0.11181	2.23
35	0.492	0.54	2.23	0.25820	1.71
$M_W/2$	0.000	0.00	0.00	1.00000	1.00

Table 1: The dependence of the various θ^* -dependent quantities on the lepton p_T .

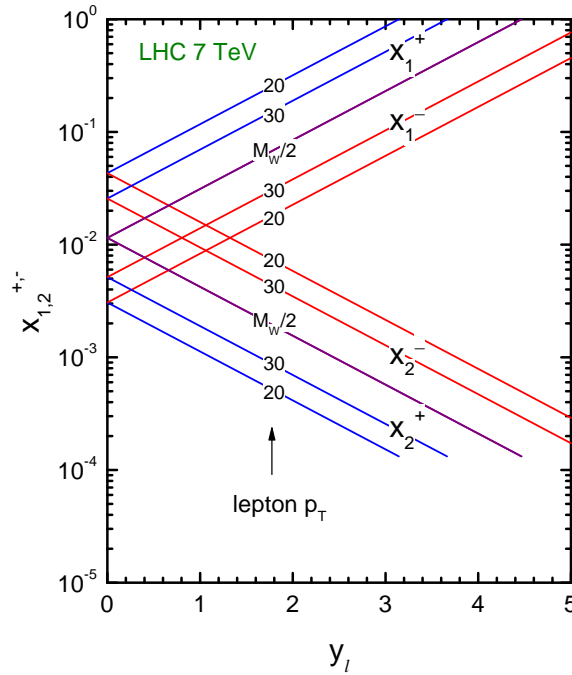


Figure 14: Dependence of x_1^\pm and x_2^\pm on y_ℓ , for lepton $p_T = M_W/2$, 30 GeV and 20 GeV, at the 7 TeV LHC. For $p_T = M_W/2$ we have $x_1^+ = x_1^-$ and $x_2^+ = x_2^-$.

respectively. The x_1^\pm and x_2^\pm values as a function of y_ℓ for three different values of lepton p_T are shown in Fig. 14.

For small or moderate y_ℓ , the x_i^+ and x_i^- contributions in (13) give comparable contributions.

⁵For massless leptons, the pseudorapidity η_ℓ is equal to the rapidity y_ℓ .

In particular, for $y_\ell = 0$,

$$x_1^+ = x_2^- = x_0 \kappa \equiv X \quad (15)$$

$$x_1^- = x_2^+ = x_0 / \kappa \equiv x \quad (16)$$

with $X/x = \kappa^2 \geq 1$. For small p_T therefore, $X \gg x$ and as long as X is not too close to 1 we may expect $V(X)\bar{q}(x) > V(x)\bar{q}(X)$, where $V(x)$ denotes either valence quark, in which case (13) becomes approximately

$$(u_V(X) - d_V(X)) \bar{q}(x) 4 \cos \theta^*, \quad (17)$$

which, in turn, leads to

$$A_\ell(0) \approx \frac{u_V(X) - d_V(X)}{u_V(X) + d_V(X) + 2\bar{q}(X)}. \quad (18)$$

Since $u_V(X) - d_V(X)$ increases with increasing X at small X , this explains, at least qualitatively, why the lepton asymmetry grows with decreasing $p_{T\min}$, see Fig. 13.

As y_ℓ increases away from 0, $x_1^+ \rightarrow 1$ and the $x_{1,2}^-$ contributions start to dominate. Furthermore, since in this region $x_1^- \gg x_2^-$, the terms with $V(x_1^-)\bar{q}(x_2^-)$ are the most important. Thus

$$A_\ell(y_\ell) \approx \frac{u_V(x_1^-)(1 - \cos \theta^*)^2 - d_V(x_1^-)(1 + \cos \theta^*)^2}{u_V(x_1^-)(1 - \cos \theta^*)^2 + d_V(x_1^-)(1 + \cos \theta^*)^2}. \quad (19)$$

According to Table 1, for $p_T \rightarrow M_W/2$, $\cos \theta^* \rightarrow 0$ and $A_\ell(y_\ell) \rightarrow A_W(y_\ell)$ because then the $(1 \pm \cos \theta^*)^2$ terms in (19) are on the same footing — the asymmetry is driven by the $u_V > d_V$ inequality which is valid at all x , and the lepton asymmetry is always positive. However for small or moderate p_T , $(1 + \cos \theta^*)^2 \gg (1 - \cos \theta^*)^2$ and so the term proportional to d_V dominates and the asymmetry is negative.

Now $d_V(x)$ decreases faster at large x than $u_V(x)$, and so at some point at large y_ℓ the approximation

$$d_V(x_1^-)\bar{q}(x_2^-)(1 + \cos \theta^*)^2 \gg u_V(x_1^-)\bar{q}(x_2^-)(1 - \cos \theta^*)^2 \quad (20)$$

breaks down, i.e. the $V \pm A$ unfavoured forward $u\bar{d} \rightarrow \ell^+ \nu_\ell$ scattering process will eventually dominate. Evidently this will happen at the y_ℓ value for which

$$u_V(x_1^-)/d_V(x_1^-) \sim (1 + \cos \theta^*)^2/(1 - \cos \theta^*)^2 = \kappa^4. \quad (21)$$

The larger the lepton p_T (recall that large p_T means small κ), the earlier (in terms of increasing y_ℓ) this will happen, as confirmed by Fig. 13. In principle LHCb data should be sensitive to this, but a very significant increase in statistics is needed compared to the present data [37], for which the MSTW2008 PDFs give a good prediction. When this is achieved then data in bins of different p_T cut will be illuminating.

In summary, the behaviour of the lepton asymmetry shown in Fig. 13 can now be understood in terms of a fairly complex interplay of PDF and $V \pm A$ effects. For small y_ℓ , the asymmetry is sensitive to the combination $u_V - d_V$ at values of x between M_W/\sqrt{s} and $\kappa(p_{T\min})M_W/\sqrt{s}$, see (18), where $p_{T\min}$ is the minimum observed p_T of the lepton and where values of $\kappa(p_T)$ are shown in Table 1. This results in a fairly clear decrease in asymmetry for increasing the minimum lepton p_T cut. At high $y_\ell > 3$ the asymmetry is even more sensitive to the minimum lepton p_T cut. In this region valence u and d quarks are being sampled at very high x , see Fig. 14.

6 Predictions for the LHC Using Modified PDFs

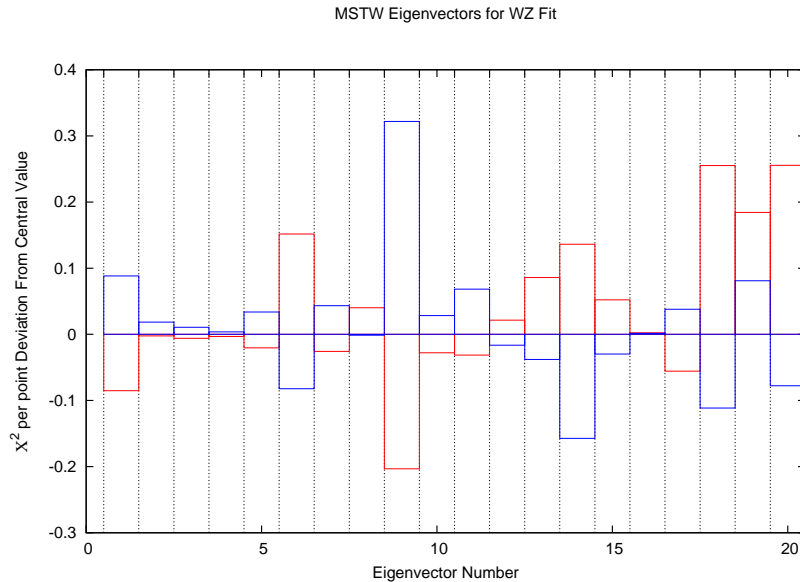


Figure 15: The variation in the quality of the fit to ATLAS vector boson production as a function of rapidity for different MSTW2008 eigenvectors. Eigenvectors 9, 14 and 18 are mainly associated with the gluon, d_V and u_V respectively.

The MSTW2008 set of PDFs are found to give very good predictions for the vast majority of relevant measurements made at the LHC to date. However, an exception is the charged lepton asymmetry arising from W^\pm production, where it is clear that this set of PDFs does not give the optimum description [15–17, 38]. In this Section we investigate the reason for this deficiency. We will show that the extended ‘Chebyshev’ parameterisation and, to a lesser extent, the improvement in the treatment of the shadowing corrections to the deuteron data, completely remove the problem, without a deterioration in the excellent description of the other data.

6.1 Preparatory study of LHC lepton asymmetry

However, first let us start by describing a recent quantitative study [17] of the CMS [38] and ATLAS [15] measurements on the *charged lepton asymmetry* with cuts on charged lepton p_T of 25 GeV and 20 GeV respectively, with a cut on missing energy of 25 GeV in the latter case. In this paper [17], a *reweighting* technique, originally introduced in [33, 39] with a slightly different method of application, was used to estimate the effect these new asymmetry data sets would have on both the central value and uncertainty of the MSTW2008 PDF set. The reweighted PDFs were able to turn an initial χ^2 of 2-3 per asymmetry data point into just over 1 per point. As is clear from the previous Section, the major PDF sensitivity is in the $u_V - d_V$ distribution. When these new data sets were included, $u_V - d_V$ was found to change most for $x = 0.02$ for $Q^2 = 10^4 \text{ GeV}^2$, increasing by about 5%, which is similar to the size of its uncertainty. Due to the sum rule, this also resulted in a reduction for $x < 0.001$, a region where there is no

data constraint. The reduction is also about the size of the uncertainty. After the reweighting, the uncertainty reduced to about 60 – 70% of its original size near $x = 0.02$. The data from the higher luminosity CMS measurement [16], obtained with a higher minimum lepton p_T cut of 35 GeV, were not studied in [17], but it is clear from the comparison in [16] that their description using MSTW2008 PDFs is worse than the data with lower p_T cut.

Here we extend this previous study somewhat. We begin by comparing, at NLO for simplicity, the fit quality to the *full* ATLAS lepton rapidity data from [15], of which the asymmetry measurements are only a subset, and where information on the size and shape in rapidity of the cross section is lost when taking the ratio of the difference to the sum of W^+ and W^- cross sections. Comparing the full data set to the MSTW2008 PDFs at NLO (without higher order electroweak corrections), using APPLgrid [40], we calculate $\chi^2/N_{\text{pts.}} = 60/30$, where the 30 data points are 11 each from W^\pm production and 8 from Z production. As for the asymmetry data, it is clear that MSTW2008 PDFs do not provide the optimum fit quality, but we note that the PDF sets of all other groups (for which APPLgrid can easily be used to calculate cross section predictions) seem to also obtain $\chi^2/N_{\text{pts.}}$ noticeably more than 1 per point, the best being the CT10 PDF set, with about 1.1 per point⁶. In order to investigate the manner in which the description of these data could be improved we look at the quality of the χ^2 using each of the uncertainty eigenvectors. This is shown in Fig. 15. The fit quality to the ATLAS data improves markedly, i.e. by about $0.2 \times 30 = 6$ units, in one direction for eigenvector 9, which is mainly associated with the gluon distribution, so this alters the common shape and normalisation of all three (W^\pm , Z) rapidity distributions via the dependence of the quarks at high Q^2 on the gluon due to evolution. The value of χ^2 improves by slightly less for eigenvectors 14 and 18. These are associated with d_V and u_V respectively; variation in these affects the asymmetry. Due to the fact that we underestimate the asymmetry, it is clear that the eigenvector directions, leading to the improvement, decrease d_V or increase u_V in the region of $x = 0.02$. Hence it follows that the fit to these data sets would move the PDFs towards these directions for u_V and d_V (and also modify the gluon distribution to some extent).

This conclusion is verified by reweighting the PDFs according to the prescription in [17], with asymmetric PDF uncertainties, using the full data on the W and Z rapidity distributions. The result for the difference in the valence quarks, $u_V(x) - d_V(x)$ at $Q^2 = 10^4 \text{ GeV}^2$, is shown in the upper of Fig. 16, compared to the MSTW2008 PDFs. The study uses 1000 random PDF sets. After reweighting, the effective number of sets $N_{\text{eff}} = 190$. The small fraction ($\sim 20\%$) of effective PDFs arising in the reweighting procedure shows that there is a significant variation in the fit quality for different random sets; some give enhanced quality compared to the central MSTW2008 set but many give rather worse quality. There is a distinct tendency for $u_V - d_V$

⁶We note that the particularly good description by CT10 is probably due to the larger strange distribution in their PDF set than in the others. A study by ATLAS has shown that their data prefer a large strange distribution, in fact seemingly one which is the same size as the \bar{u} and \bar{d} distributions even at low Q^2 [41]. We do indeed see some small improvement in fit quality associated with the eigenvector most associated with the strange PDF normalisation, but rather less than for the three eigenvectors 9, 14, 18 mentioned below. This means that only a marginal improvement can be obtained by changing the strange distribution by one standard deviation. We have confirmed this by making a more thorough study. Moreover, a study by the NNPDF group has reached a similar conclusion [6].

to increase at $x \approx 0.02$, and correspondingly decrease at lower x . However, the effect is less pronounced than that seen in [17] when using only the asymmetry data, with the change in the average value being less than the uncertainty, even at $x = 0.02$. This slightly smaller change has two origins. Firstly the small systematic uncertainties are treated as entirely uncorrelated in the asymmetry, but some history of the correlation persists in the full treatment of the separate W^+ and W^- rapidity distributions. For example, one correlated systematic moves the W^+ and W^- rapidity distribution up and down in opposite directions, independent of rapidity, and clearly contributes to some correlation in the asymmetry. Maintaining this information allows for a slightly easier fit to the asymmetry. Secondly, if only asymmetry data are used, all PDFs which carry a high weight must improve the comparison for the asymmetry. If the full data are used, some higher weight PDFs produce better fits due to improvement in overall shape of all distributions with rapidity, or improvement in consistency of W and Z data, so not all high weight PDFs have an increase in $u_V - d_V$ at $x \sim 0.02$. This shows that comparing to asymmetry data alone can exaggerate their effect and importance. In the lower of Fig. 16 we also show the effect of the new data on the gluon distribution. The change is not large, but is not entirely insignificant, and an improvement in the shape of the rapidity distributions does require a modification of the gluon distribution. After reweighting the fit quality improves to $\chi^2/N_{\text{pts.}} = 48/30$.

6.2 LHC W, Z rapidity distributions from modified MSTW PDFs

We now come to an important result: that is, the description of the ATLAS W^\pm, Z rapidity data at NLO using the two modified PDF sets extracted earlier in this paper. Namely the PDF set based on the Chebyshev polynomial parameterisation (MSTW2008CP), and the set ‘MSTW2008CPdeut’ including the improved deuteron corrections in addition. The change in u_V in MSTW2008CP is actually rather similar to that for eigenvector 18, though bigger, and the further change in MSTW2008CPdeut is similar to eigenvector 14. As one would expect, this does lead to an improvement in the comparison to the data. The $\chi^2/N_{\text{pts.}}$ improves from 60/30 to 49/30 for MSTW2008CP, and to 46/30 for MSTW2008CPdeut. This is as good as any other PDF set at NLO, except for CT10 which has a much larger strange quark distribution than that of other global analyses (as discussed in the previous footnote). For the ATLAS asymmetry data the $\chi^2/N_{\text{pts.}}$ improves from 30/11 to 15/11 for MSTW2008CP to 9/11 for MSTW2008CPdeut.

It is particularly informative to study how the combination $u_V - d_V$ is changed in the new fits with the extended parameterisations. Fig. 17 compares the values of $u_V - d_V$ obtained in the MSTW2008CP and MSTW2008CPdeut fits with those of the original MSTW2008 analysis. Indeed, $u_V - d_V$ increases dramatically in the region $x \sim 0.01 - 0.06$ which is probed by the W^\pm, Z rapidity distributions at the LHC. We emphasise that exactly the same data sets are used in all three analyses. Although some Tevatron lepton asymmetry data were included in the original MSTW2008 analysis, it is remarkable that the extended parameterisations make such a sizeable change to $u_V - d_V$ for $x \sim 0.01 - 0.06$ and improve the overall fit to the data, as well as giving a good description of the LHC lepton asymmetry data (which were not included in the fits). Moreover, the changes in u_V, d_V are very small in the region $x > 0.05$ where they are constrained by other types of data in the fits. An exception is the approximately 5% increase

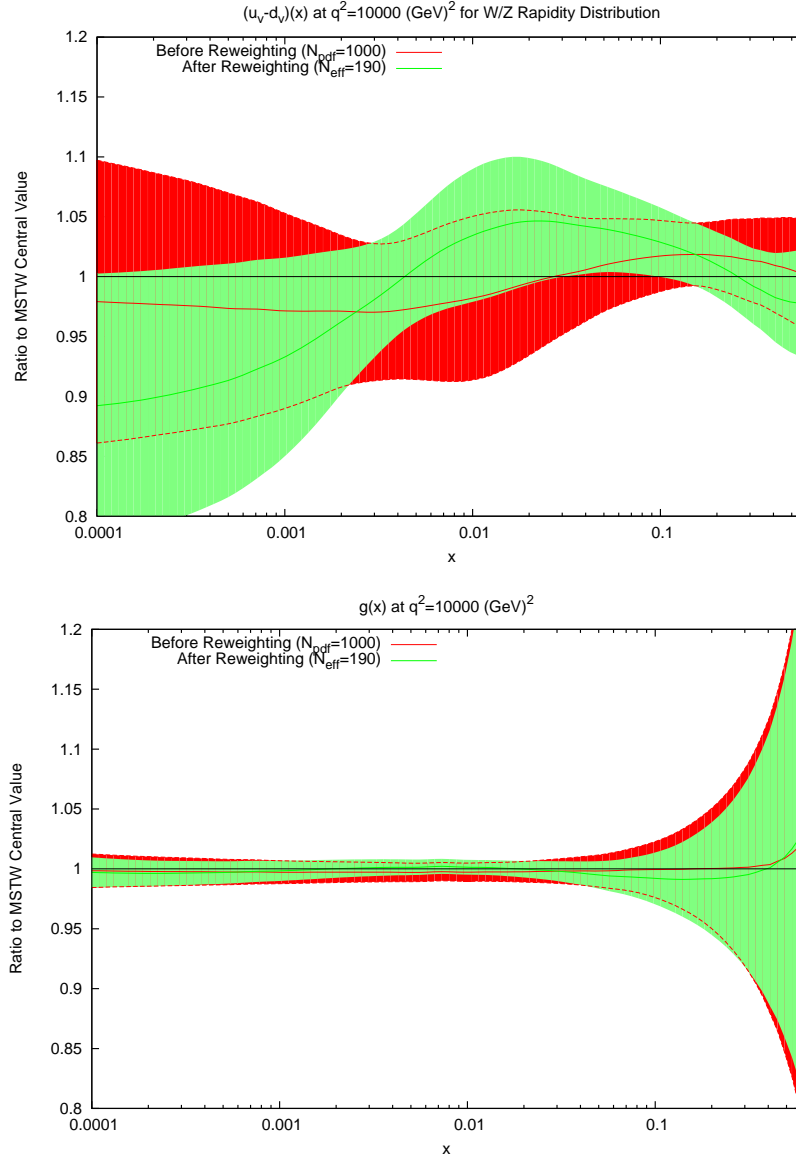


Figure 16: The effect of reweighting on the $u_V(x) - d_V(x)$ distribution (top) and $g(x)$ distribution (bottom) at $Q^2 = 10^4 \text{ GeV}^2$. The darker (red) band shows the average value and standard deviation of 1000 randomly generated sets of MSTW2008 PDFs and the lighter (green) band the same PDFs after reweighting according to the fit quality for the ATLAS data on W, Z rapidity.

in d_V for $x \sim 0.5$ when the deuteron corrections are included. For $x \lesssim 0.01$, where there are no data constraints, and which has little impact on the LHC lepton asymmetry, the changes can be extremely large – ranging from an increase of up to 50% at $x = 0.005$ to a significant decrease below $x \sim 0.0005$.

The large change in u_V and d_V in the extended ‘Chebyshev’ parameterisations in the region $x \lesssim 0.01$ unconstrained by data, has an interesting consequence. The small x behaviour of the

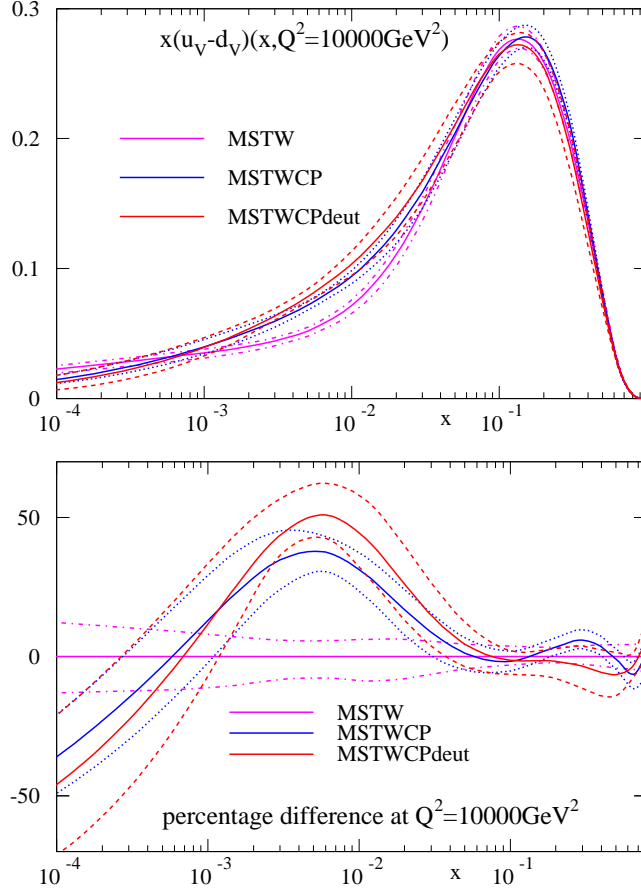


Figure 17: The combination $u_V - d_V$ obtained in the new MSTW2008CP and MSTW2008CPdeut fits compared with that of the original MSTW2008 analysis. All three analyses fit to exactly the same data sets.

input valence distributions in the original NLO MSTW2008 parameterisation was controlled by the parametric forms:

$$xu_V \propto x^{\delta_u} \quad \text{with} \quad \delta_u = 0.29 \pm 0.02, \quad (22)$$

$$xd_V \propto x^{\delta_d} \quad \text{with} \quad \delta_d = 0.97 \pm 0.11, \quad (23)$$

at $Q_0^2 = 1 \text{ GeV}^2$. On the other hand with the ‘Chebyshev’ parameterisations we have $\delta_u = 1.00$ and $\delta_d = 0.70$ for the MSTW2008CP fit, and $\delta_u = 0.70$ and $\delta_d = 0.65$ for the MSTW2008CPdeut fit, where in each case the uncertainties are a little bigger than for the MSTW2008 set. This is much more in line with the Regge expectation that the two powers should be the same, particularly in the case of MSTW2008CPdeut, where the difference is easily consistent with zero within uncertainties. The powers are also fairly close to the Regge expectation $\delta \sim 0.5 - 0.6$. It seems as though the standard parameterisation for the MSTW2008 valence quarks, combined with the constraint from a variety of data, pushes the small- x valence quarks in a

direction somewhat at odds with the LHC asymmetry data. Less constraint from other data, and potentially an equally restrictive, but different, parameterisation, possibly with the small- x behaviour of d_V and u_V tied together more closely, may have provided a better prediction. However, the extended parameterisation, introduced here, seems to be a preferable approach, describing all data sensitive to u_V and d_V well and automatically making the small- x forms of u_V and d_V more similar.

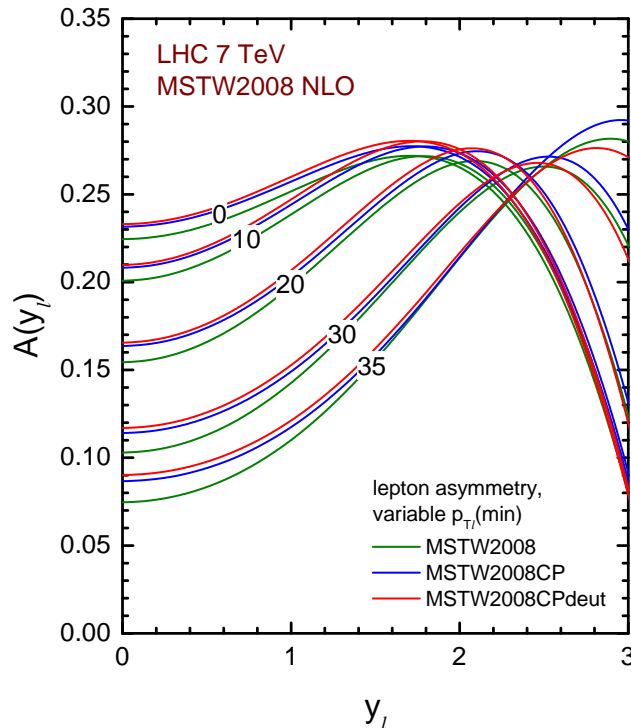


Figure 18: The variation in the predictions for lepton asymmetry data calculated at LO and zero width using the original and the two modified MSTW2008 PDFs. The sets of curves correspond to different choices of the minimum p_T cut (shown in GeV) applied to the observed charged lepton from the W decay.

Let us discuss the features of Fig. 17, and the description of the charged lepton asymmetry, in a little more detail. The change in the u_V and d_V distributions in MSTW2008CP and MSTW2008CPdeut seems ideal for improving the comparison to the ATLAS lepton rapidity data, and for removing the shortcomings in the description of these data by the MSTW2008 set of PDFs. However, it can be noted that the apparent difficulties in the MSTW description of the LHC lepton asymmetry data are very much correlated with the minimum p_T cut applied to the final state lepton. The comparisons with unpublished data obtained with a $p_T = 20$ GeV cut seemed perfectly good (see Fig. 9 of [45, 46]), but that for data obtained with a higher $p_T = 35$ GeV cut were much worse [16]. From the study in the previous Section, we see that changes in the minimum p_T cut on the data change the x range probed in $u_V - d_V$. As $p_{T\min}$

decreases, the x region expands to include lower and lower values of x . Hence, the change that MSTW2008CP and MSTW2008CPdeut make to our predictions for the lepton asymmetry are very dependent on the choice of $p_{T\min}$ of the corresponding data. This is shown explicitly in the predictions shown in Fig. 18 (where, as in the previous Section, the curves are made using LO formulae and NLO PDFs). We see that the use of MSTW2008CP PDFs increases the lepton asymmetry at low rapidity, more so for a higher minimum p_T cut. That is, the proportional change near $y_\ell = 0$ for $p_T = 35$ GeV is about 15%, whereas for $p_T = 20$ GeV it is only about 5%. The asymmetry at low rapidity increases slightly further when using MSTW2008CPdeut, but by only a small amount compared to the change resulting from using MSTW2008CP PDFs. Hence, the majority of the change is obtained from an extension in the PDF input parameterisation, and only a minor amount when flexible deuteron corrections are also included. We note from the figure that for the highest p_T cuts the asymmetry for $y_\ell \gtrsim 2.5$ decreases for MSTW2008CPdeut due to the larger value of d_V for $x \sim 0.5$.

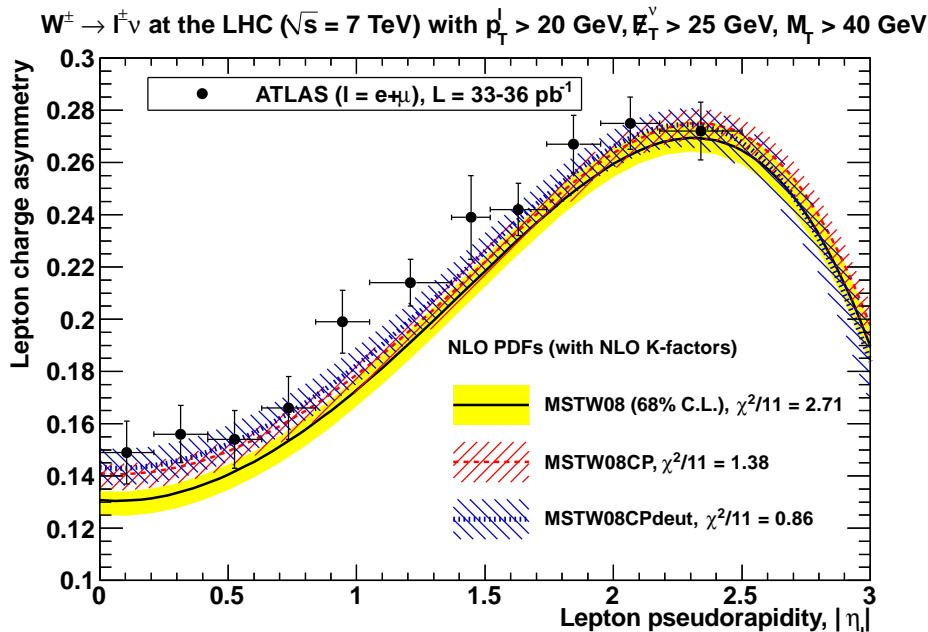


Figure 19: The improvement in the quality of the prediction of the ATLAS lepton asymmetry data for $p_T > 20$ GeV (and missing transverse energy $E_T^\nu > 25$ GeV) [15], in going from the original MSTW2008 \rightarrow MSTW2008CP \rightarrow MSTW2008CPdeut sets of partons. All three parton sets are obtained by fitting to exactly the same (pre-LHC) data set.

The detailed NLO comparisons to the ATLAS asymmetry data for $p_T > 20$ GeV (and missing transverse energy $E_T^\nu > 25$ GeV) and CMS electron asymmetry data for $p_T > 35$ GeV (made using NLO K -factors [17] computed with DYNNLO [47]) are shown in Fig. 19 and Fig. 20. The latter is the data set most sensitive to the valence quark difference at small x and which is predicted worst by MSTW2008 PDFs. One sees that the initial poor χ^2 per point of 5.3 for MSTW2008 is reduced to 1.5 for MSTW2008CP, and to slightly less than 1 for MSTW2008CPdeut, where all

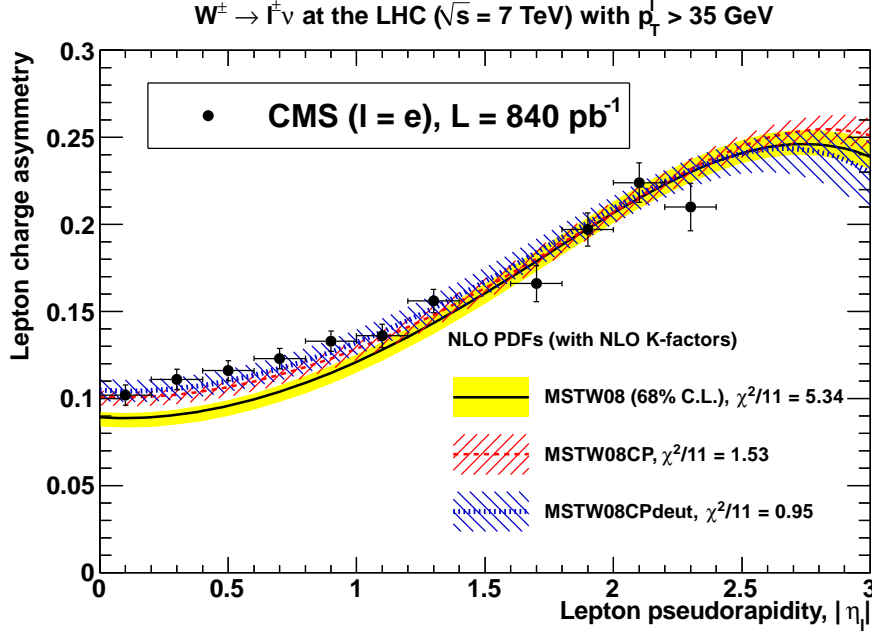


Figure 20: The improvement in the quality of the prediction of the CMS lepton asymmetry data for $p_T > 35$ GeV [16], in going from the original MSTW2008 \rightarrow MSTW2008CP \rightarrow MSTW2008CPdeut sets of partons. All three parton sets are obtained by fitting to exactly the same (pre-LHC) data set.

experimental uncertainties are simply added in quadrature. The last could not be a much better description of the data. The improvement is similar for the ATLAS data, but the standard MSTW2008 PDFs do not give such a poor prediction in this case. Note that these are all *predictions*; neither these data, nor indeed any other LHC data have been used in order to extract the PDFs. The main reason for the small extra change coming from the MSTW2008CPdeut PDFs is simply due to the removal of the significant small- x shadowing deuteron correction in the default MSTW2008 extraction – recall that our freely determined deuteron correction is extremely small for $x \sim 0.02$. The uncertainty band for the prediction for MSTW2008CP is very similar to that for MSTW2008. The extended parameterisation has changed the average value of $u_V - d_V$ far more than it affects the nominal uncertainty. The uncertainty band for MSTW2008CPdeut is a little bigger, reflecting the extra uncertainty introduced by having a varying deuteron correction.

To conclude this subsection, we also examine the effect of the W and Z rapidity data on the MSTW2008CP and MSTW2008CPdeut PDFs by looking at the eigenvector sensitivity and using the *reweighting procedure*. The change in χ^2 for each the eigenvectors of the MSTW2008CP set is shown in Fig. 21. The dominant eigenvector, number 12, is still mainly to do with the gluon, but some variation of the strange quark is mixed in. The situation is very similar for the MSTW2008CPdeut set. Hence, we still obtain a small effect on the gluon distribution similar to that for MSTW2008 shown in the lower of Fig. 16. However, even with the modified sets there

are still some small changes required for the $u_V - d_V$ distribution, as shown in Fig. 22. After reweighting the fit quality improves to $\chi^2/N_{\text{pts.}} = 39.5/30$ for MSTW2008CP and $\chi^2/N_{\text{pts.}} = 38.5/30$ for MSTW2008CPdeut. Note however, that the effective number of PDFs is far greater than in the case for MSTW2008, showing the increased compatibility of the data and the PDFs. Even though some of the eigenvectors that show an improved fit to the data are those with a larger strange quark at small x , the amount of weight the random PDFs with larger strange fraction carry is not such to show a clear increase in the strange fraction after reweighting. Essentially the direct constraint from the dimuon data is overwhelming any pull from the ATLAS data.

Hence the MSTW2008CP and MSTW2008CPdeut PDFs result in a big change in the high- p_T cut, low-rapidity lepton asymmetry at the LHC. However, as we have shown, this quantity is extremely specifically sensitive to $u_V - d_V$ for $x \sim M_W/\sqrt{s}$, the PDF combination that changes by far the most.

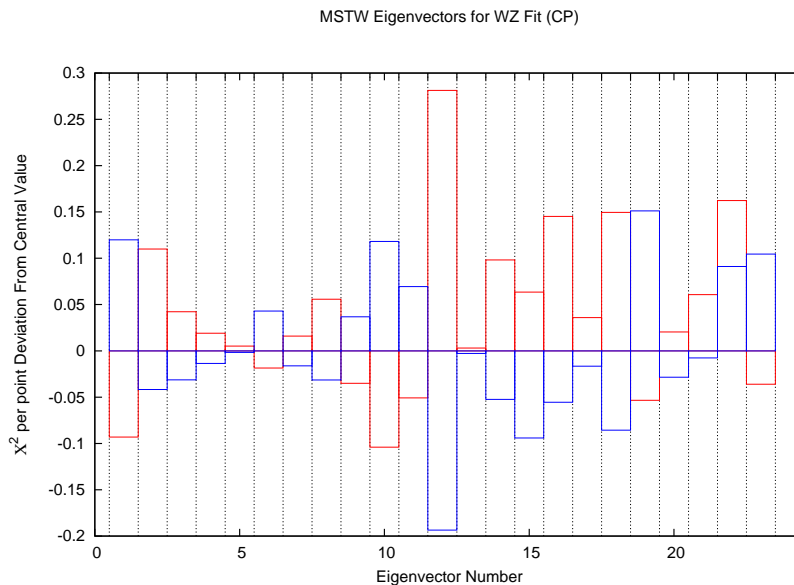


Figure 21: The variation in the quality of the fit to ATLAS vector boson (W, Z) production as a function of rapidity for different MSTW2008CP eigenvectors.

6.3 Predictions for other LHC data using modified MSTW PDFs

What about the predictions for other LHC observables? Apart from u_V, d_V at low x , other PDFs have changed little, especially the gluon distribution which hardly changes at all from MSTW2008 compared to the size of its uncertainties. Similarly, in the new fits $\alpha_S(M_Z^2)$ is left free, but only experiences a tiny change. Hence, we would expect little variation in most cross section predictions, as compared to those of the MSTW2008 PDFs. This is verified in Table 2 where we show the percentage variation in predictions for various standard cross sections compared to those using MSTW2008 PDFs (the Higgs boson predictions are for $M_H = 125$ GeV). We see that there is extreme stability in the total cross sections predictions. All the changes

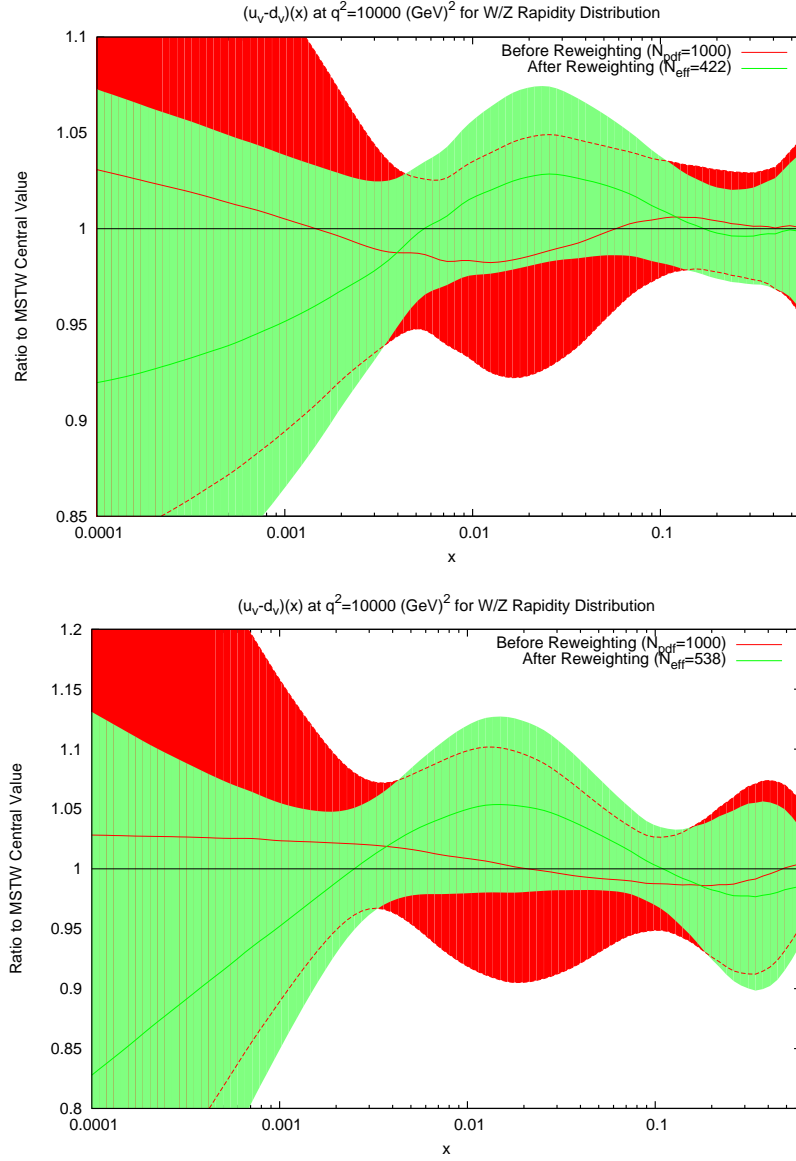


Figure 22: The effect of reweighting on the $u_V(x) - d_V(x)$ distribution at $Q^2 = 10^4 \text{ GeV}^2$. The darker (red) band shows the average value and standard deviation of 1000 randomly generated sets of MSTW2008CP (top) and MSTW2008CPdeut (bottom) PDFs and the lighter (green) band the same PDFs after reweighting according to the fit quality for the ATLAS data on W, Z rapidity.

are inside the uncertainties, and in most cases by very much less than the uncertainty. Even $\sigma(W^+)/\sigma(W^-)$ changes by barely more than 1%, reflecting the fact that the largest change in the asymmetry is for a small region of phase space, i.e. high- p_T and low- y_ℓ and, moreover, is rather small compared to the individual cross sections. Hence, the excellent agreement of predictions using MSTW2008 PDFs with the measurements of W and Z total cross sections by ATLAS [15] and CMS [42] is not altered. The change is also displayed in Fig. 23, for a continuous range of

	CP	CPdeut	unc.
W Tevatron (1.96 TeV)	+0.6	+0.1	1.8
Z Tevatron (1.96 TeV)	+0.8	+0.7	1.9
W^+ LHC (7 TeV)	+0.7	+0.3	2.2
W^- LHC (7 TeV)	-0.7	-0.4	2.2
Z LHC (7 TeV)	+0.0	-0.1	2.2
W^+ LHC (14 TeV)	+0.6	+0.3	2.4
W^- LHC (14 TeV)	-0.6	-0.5	2.4
Z LHC (14 TeV)	+0.1	-0.1	2.4
Higgs Tevatron	-0.5	-1.8	5.1
Higgs LHC (7 TeV)	+0.2	-0.1	3.3
Higgs LHC (14 TeV)	+0.1	+0.1	3.1
$t\bar{t}$ Tevatron	+0.5	-0.6	3.2
$t\bar{t}$ LHC (7 TeV)	-0.4	-1.8	3.9
$t\bar{t}$ LHC (14 TeV)	-0.2	-0.8	3.1

Table 2: The percentage change of various cross sections due to the modifications of the MSTW2008 PDFs. CP denotes the fit with the Chebyshev polynomial input parameterisation, and CPdeut denotes the fit with the deuteron corrections included in addition. To demonstrate the small changes in the cross sections, we also show, in the final column, the symmetrized PDF $+\alpha_S(M_Z^2)$ percentage uncertainties for the MSTW2008 PDFs.

LHC collider energy. This illustrates the same results. There is a reasonable change of $\sim 1.5\%$ in the $t\bar{t}$ cross section for the lowest LHC energies, but this is where the gluon distribution is probed at relatively high x , where the uncertainties are largest. The PDF uncertainty on $\sigma_{t\bar{t}}$ for 7 TeV is 2.9%, or 3.9% when the $\alpha_S(M_Z^2)$ uncertainty is also included. As smaller x values are probed at higher energy the uncertainty reduces, e.g. to 3.1% at 14 TeV, and the difference between the MSTW2008 and MSTW2008CPdeut predictions also reduces to less than 1%.

Finally, we consider the description of LHC jet data. The χ^2 is defined as

$$\chi^2 = \sum_{i=1}^{N_{\text{pts.}}} \left(\frac{\hat{D}_i - T_i}{\sigma_i^{\text{uncorr.}}} \right)^2 + \sum_{k=1}^{N_{\text{corr.}}} r_k^2, \quad (24)$$

where $\hat{D}_i \equiv D_i - \sum_{k=1}^{N_{\text{corr.}}} r_k \sigma_{k,i}^{\text{corr.}}$ are the data points allowed to shift by the systematic errors in order to give the best fit, $\sigma_{k,i}^{\text{corr.}}$ is an absolute correlated uncertainty and normalisation is treated as the other correlated uncertainties. The same definition is used for the comparison to W, Z data, but the definition is less important in that case. The χ^2 per point for the ATLAS inclusive jet data [43] is $\chi^2/N_{\text{pts.}} = 70/90$ for jet radius $R = 0.4$ and $\chi^2/N_{\text{pts.}} = 71/90$ for $R = 0.6$. The variation as a function of the MSTW eigenvectors for the two different choices of the jet radius is shown in the upper of Fig. 24. (The precise value of χ^2 depends on the detailed manner in which the correlated systematic uncertainties are treated.) The calculation is made at NLO using APPLgrid using a renormalisation and factorisation scale choice of $p_T^{\text{jet, max}}$

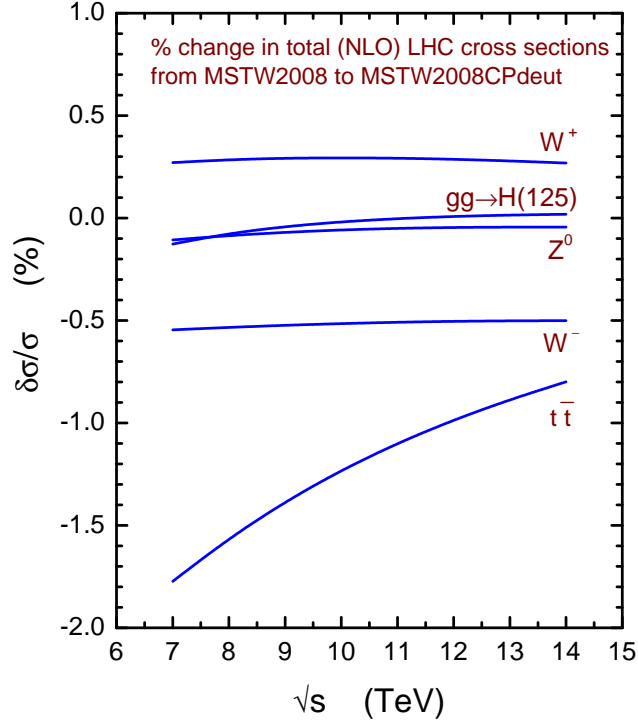


Figure 23: The variation in the prediction for various cross sections as a function of energy for the MSTW2008CPdeut PDFs compared to the values obtained from the original MSTW2008 PDFs.

in each rapidity bin, though extremely similar results are obtained using FastNLO [44]. The χ^2 for the MSTW2008 central set is very good, at least as good as the PDF sets of other groups, though the data do not discriminate strongly. Similarly the variation in χ^2 between MSTW2008 eigenvectors is very small: at the absolute most about 2 units in χ^2 for the 90 data points – though this variation is rather smaller than the few units difference observed between different PDF groups. Hence, there is even less possibility of these data improving the knowledge of a single set of PDFs than there is for them to discriminate between different sets. As one might expect, the χ^2 using MSTW2008CP and MSTW2008CPdeut is hardly changed from that for MSTW2008. In the lower of Fig. 24 we see that exactly the same picture holds for the ATLAS dijet data (which corresponds to a different analysis of basically the same data set). In this case the χ^2 values are larger, but similar between the different PDF groups, and at the very most there is 10% variation in χ^2 value between different eigenvectors, and in most cases much less. In Fig. 25 we see the effect of reweighting when using the MSTW2008 PDFs. The small variation between the random PDF sets is illustrated by the very high effective number N_{eff} of PDFs, and in both cases the change in the average value is well within uncertainties, and the decrease in the standard deviation is minimal. With reweighting the fit quality improves by only one or two units in χ^2 . Similarly the MSTW2008CP and MSTW2008CPdeut sets give a χ^2 within a couple of units of MSTW2008 for both values of R . Future data based on higher luminosity, and with a better determination of systematic uncertainties, will be more constraining and discriminating.

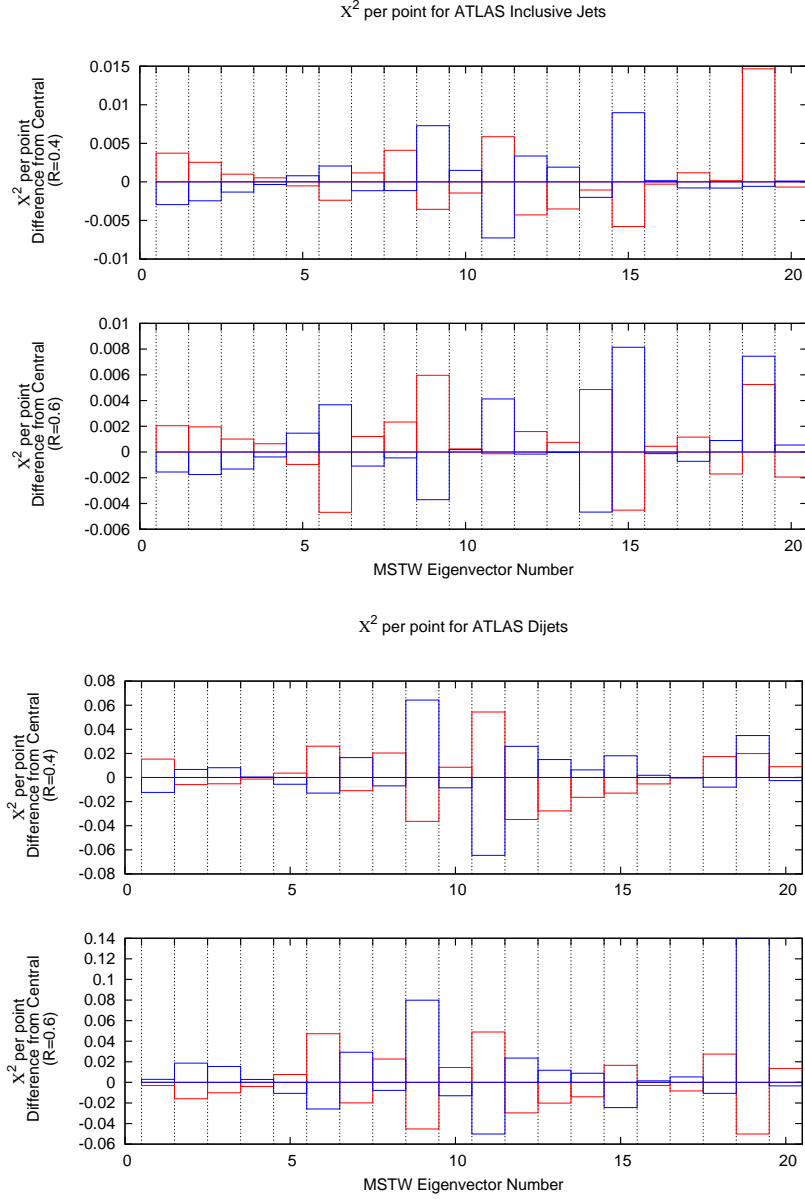


Figure 24: The variation in the quality of the fit to ATLAS inclusive jet production (top) and dijet production (bottom) as a function of rapidity for different MSTW2008 eigenvectors.

7 PDFs at NNLO

We have repeated the study of extending the parameterisation in the valence and sea quarks, and of investigating deuteron corrections, using NNLO PDFs extracted from an NNLO fit to data. We do not go into details here as all the conclusions are qualitatively similar. The input PDFs at NNLO are not exactly the same shape as at NLO due to higher-order corrections in the coefficient functions. However, as we will discuss, the relative change in the PDFs from the extended parameterisation and deuteron corrections is altered rather little by this.

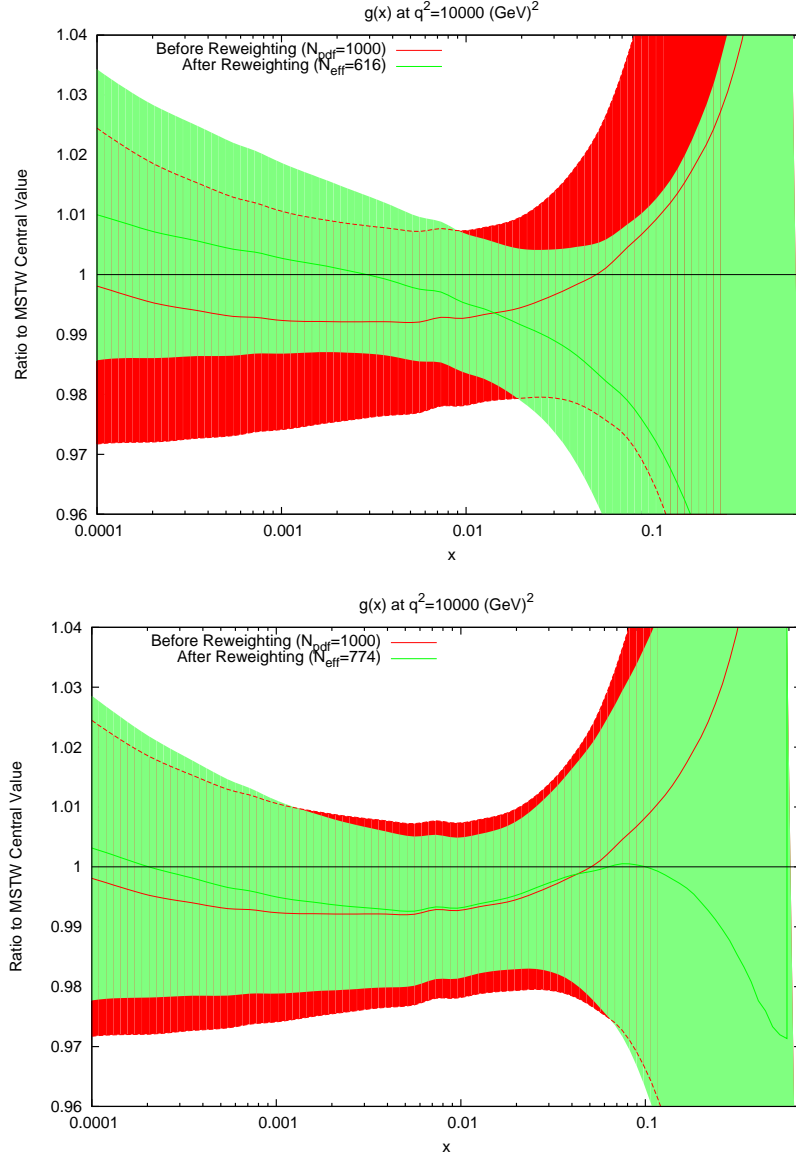


Figure 25: The effect of reweighting on the $g(x)$ distribution at $Q^2 = 10^4 \text{ GeV}^2$. The darker (red) band shows the average value and standard deviation of 1000 randomly generated sets of MSTW2008 PDFs and the lighter (green) band the same PDFs after reweighting according to the fit quality for the ATLAS data on inclusive jet production using $R = 0.4$ (top) and $R = 0.6$ (bottom).

To be a little more precise, the extended parameterisation leads to an NNLO fit with $\Delta\chi^2 = -37$ compared to the standard NNLO fit. The changes in PDFs are completely analogous, i.e. the $u_V(x)$ distribution increases for $x \sim 0.01$ and decreases at very small x , but there is little change in other PDFs. Despite the slightly larger fit improvement at NNLO, the change in $u_V(x)$ is slightly smaller than at NLO. When deuteron corrections are allowed to be free they choose a form very similar to the NLO case, though with the normalisation marginally smaller than 1 and

the precise value changing slightly if the pivot point x_p is varied. If $x_p = 0.03$ is used instead of $x_p = 0.05$ the fit is only three units worse, but the deuteron correction is very similar to that at NLO, with normalisation marginally above 1. The fit has $\Delta\chi^2 = -82$ compared to our default fit. As at NLO the only additional change in PDFs is another slight change in $u_V(x)$ and a similar change in $d_V(x)$ to that at NLO. Again the changes are qualitatively the same to those at NLO, but slightly smaller. The changes to predicted cross sections (such as those shown in Table 2 and Fig. 23) using the modified NNLO PDFs are very similar to those at NLO, perhaps a little smaller in general, and as at NLO are much smaller than the PDF uncertainties.

The changes in the modified NNLO PDFs automatically significantly improve the deficiencies that MSTW2008 PDFs have with the LHC asymmetry. The effect from the PDFs is a little less than at NLO due to the smaller change. However, this is well within uncertainties, and is, for example, no longer true for the $x_p = 0.03$ fit mentioned above. (Similarly the agreement of the small- x powers δ for the up and down valence distribution are in better agreement for $x_p = 0.03$ – the values of δ being very sensitive to small changes in detail.) We also note that the NNLO cross section corrections themselves automatically improve the description of data very slightly. As at NLO, the best fit to the total W, Z rapidity data can still be improved, and at NNLO the fit to the general shape in rapidity is a little worse than at NLO. While the modified PDFs essentially cure the problem with the asymmetry, detailed changes in the gluon and sea are required for the best possible fit as shown by the reweighting exercise at NLO described above.

8 Conclusions

In this paper we have performed global PDF analyses in which the standard MSTW input parameterisations of the PDFs have been made more flexible by replacing the usual $(1 + \epsilon x^{0.5} + \gamma x)$ factors in the valence, sea and gluon distributions by Chebyshev polynomial forms $(1 + \sum a_i T_i(y))$, with $y = 1 - 2\sqrt{x}$. A Chebyshev form has the advantage that the parameters a_i are well-behaved and, compared to the coefficients in our standard parameterisation, are rather small, with moduli usually ≤ 1 . We demonstrated that four Chebyshev polynomials are sufficient for high precision and used this number in the valence and sea distributions. However, the gluon distribution, which already had seven parameters, did not require extra free parameters. Hence, only two Chebyshev polynomials were used in this case, which is equivalent to the usual $(1 + \epsilon x^{0.5} + \gamma x)$ factor.

To explore the effects of using these more flexible input forms we fit to exactly the same data set as was used for the MSTW2008 analysis [1]. The resulting parton set was called MSTW2008CP. We found some improvement in the fit to the data, but the only significant PDF change was in the valence up-quark distribution, u_V , at small x . Although left free in the fit, the change in $\alpha_S(M_Z^2)$ is tiny.

The use of Chebyshev forms allows a more consistent determination of the uncertainties of the PDFs. In our best fit we have 6 more free parameters than in the original MSTW2008 fit. In the determination of the uncertainties we allow one extra parameter to be free for the u_V , d_V and sea quark PDFs when evaluating the uncertainty eigenvectors. Hence, in the determination

of the uncertainties of the MSTW2008CP PDFs we have 23 eigenvectors, rather than the 20 eigenvectors of the MSTW2008 analysis. Despite having extra eigenvectors, the uncertainties were found to be similar to those of the MSTW2008 partons, but tend to be larger, particularly for valence quarks, at small x . The most significant change is in u_V which now has a more realistic uncertainty, without the artificial ‘neck’ for $x \sim 0.003$ at $Q^2 = 10000 \text{ GeV}^2$.

We also performed a detailed investigation of the nonperturbative corrections to be applied when fitting to the data obtained from deuteron targets. It will be important to continue to include these deuteron data in global PDF analyses for the foreseeable future, in order to separate the u_V and d_V PDFs. The deuteron correction factor was parametrised in terms of 4 variables, and various MSTW2008CP global fits were performed allowing some, or all, of these variables to be free. We found that large improvements could be obtained in the description of the deuteron data, and also in the Tevatron charged lepton asymmetry data, as compared to the MSTW2008 analysis. The MSTW2008 fit had a fixed deuteron correction imposed, and then only at small x . Using the results from the present study, we have adopted the best, and most realistic, deuteron correction. The corresponding parton set is called MSTW2008CPdeut. The most significant change, in comparison to MSTW2008CP, is in the d_V PDF, and in the uncertainties of both valence PDFs. Again the change in $\alpha_S(M_Z^2)$ is insignificant.

In summary, the main changes to the MSTW2008 PDFs obtained in the CP and CPdeut ‘Chebyshev’ fits are in u_V and d_V for $x \lesssim 0.03$ at high $Q^2 \sim 10000 \text{ GeV}^2$, or slightly higher x at low Q^2 : a region where there are few or weak constraints on the valence PDFs from the data used in these fits. There is also an approximately 5% increase in d_V for $x \sim 0.5$ in the CPdeut fit.

We have drawn attention to one type of measurement that is particularly sensitive to u_V and d_V in the small x region. That is the decay charged lepton asymmetry from W^\pm production, which probes a little more deeply into the small x region as the collider energy is increased. For a 7 TeV collider, the probed region is $0.01 \lesssim x \lesssim 0.05$. The PDF combination sampled by the asymmetry is $(u_V - d_V)$ at low lepton rapidities. However, we showed that the description of the asymmetry data has a more intricate dependence on the PDFs at larger rapidities, and also depends sensitively on the experimental minimum p_T cut applied to the observed lepton. In Fig. 17 we plotted $(u_V - d_V)$ at the scale $Q^2 = 10^4 \text{ GeV}^2$ approximately relevant for W, Z production, obtained from the original MSTW2008 analysis together with the behaviour coming from the CP and CPdeut PDF sets; all three sets were fitted to exactly the same (pre-LHC) data.

In Figs. 19 and 20 we showed the predictions for the ATLAS and CMS asymmetry measurements obtained from the three sets of PDFs. It is remarkable that the ‘Chebyshev’ sets, and, in particular the MSTW2008CPdeut set, give such excellent descriptions of these data, which were not well predicted at low rapidities by the original MSTW2008 set. Note that the improvement is due to using a more flexible and more physically suitable (‘Chebyshev’) parameterisation of the input PDFs, and to a lesser extent, to taking more care with the deuteron corrections. We emphasise again that the main changes to the MSTW2008 PDFs are in the valence distributions in the small x region which is barely probed by the existing data. Lepton asymmetry at higher LHC energies will sample this region more and at smaller x values. It is not surprising, therefore, that we found that the predictions of the original MSTW2008 PDFs are essentially unchanged for all other observables. In all the cases that we investigated, the changes in the cross sections,

compared to those of MSTW2008, were much smaller than the PDF uncertainties. This was even the case for the total $\sigma(W^+)/\sigma(W^-)$ ratio, since each cross section only obtains a small contribution from the high- p_T cut, low- y_ℓ region, and moreover the change in the PDFs does not even change the individual differential cross sections by very much, but in a manner which is maximised in the asymmetry measurement. Hence, for the overwhelming majority of processes at the LHC the MSTW2008 PDFs give essentially the same result as those resulting from the investigations in this article.

From the results of this paper it is clear that a full update of MSTW2008 PDFs would benefit from an extended parameterisation similar to the form presented, and a modification of the deuteron corrections of some sort, together with some account of the uncertainty associated with these. Moreover, it would require other features, most particularly an inclusion of new data, including the LHC data considered in this article. However, from the preliminary studies already undertaken of including new data in [10, 17] and in this article, it is clear that there are no signs of very significant changes other than the form of the valence quarks at small x .

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